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THE UNIVERSITY OF ALBERTA

RECTANGULAR PRESTRESSED CONCRETE BEAMS  
SUBJECTED TO BENDING SHEAR AND TORSION

by



Rodney Neale Stark

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES  
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF  
MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

EDMONTON, ALBERTA

FALL, 1969





## ABSTRACT

## UNIVERSITY OF ALBERTA

## FACULTY OF GRADUATE STUDIES

This study is a part of investigations being carried out at the Structural Laboratory of the University of Alberta by Dr. P. Mukherjee (A) under the guidance of Dr. J. Kowarsuk (T). The results of the entire project will be presented in the form of a report at a later date.

The primary objective of this work of the investigation was to achieve a better understanding of the behavior of prestressed reinforced

The undersigned certify that they have read, and recommend to the FACULTY OF GRADUATE STUDIES for acceptance, a thesis entitled "RECTANGULAR PRESTRESSED CONCRETE BEAMS SUBJECTED TO BENDING, SHEAR AND TORSION", submitted by Rodney Neale Stark in partial fulfilment of the requirements for the degree of Master of Science.





## ABSTRACT

This study followed a continuing program of investigations being carried out at the Structural Laboratory of the University of Alberta by Dr. P. Mukherjee (\*) under the guidance of Dr. J. Warwaruk (+). The results of the entire program will be presented in the form of a report at a later date.

The primary objective of this phase of the investigation was to achieve a better understanding of the behavior of pretensioned reinforced rectangular concrete beams subjected to torsion, shear and flexure.

Twenty four beams having a nominal cross section of 6 x 12 in. and containing identical amounts of mild steel reinforcement were tested. Two levels of prestress were studied; each level consisting of six beams concentrically prestressed and six more eccentrically prestressed.

The testing equipment used for this investigation permitted independent application of the twisting moment and transverse loads. The ratio of twisting moment to flexural moment was varied for each type of prestressing. All beams were tested to failure by applying the loading in a series of predetermined increments. The test results are presented in the form of tables, graphs, and interaction diagrams.

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## CHAPTER I

### INTRODUCTION

#### 1-1 INTRODUCTORY REMARKS

Torsion has generally been considered a secondary effect in reinforced and prestressed concrete structures. Consequently, it has long been neglected compared with other areas of concrete technology. Only recently has torsion become an increasingly important factor in structural design. Refinements in analysis, the greater use of new structural shapes by architects, and the employment of ultimate strength rather than working stress design have brought about the need to study torsion in the same detail as bending and shear.

The behavior of concrete members subjected to flexure, shear, prestress, and varying combinations of these effects has been investigated quite thoroughly. However, the study of such members subjected additionally to torsion is a relatively new field of research. From such studies, the behavior of concrete under all possible loading conditions will be more fully understood. Once having filled these gaps in our knowledge, more modern and aesthetic structures will be designed and constructed in the future.

#### 1-2 OBJECT

The primary object of this investigation was to study the behavior of reinforced prestressed rectangular concrete beams subjected to





torsion, shear and flexure. The variables studied included both the level and type of prestress, and the beams were subjected to varying ratios of torsional moment to bending moment. For each beam, all strands were to be stressed to the same level of prestress. Elastic shortening of the concrete and relaxation in the strands were measured to enable calculation of an effective prestress force at the time of testing. The behavior of both the transverse and longitudinal reinforcement was observed using electrical resistance strain gauges positioned in appropriate locations.

All the beams were fabricated and tested according to the procedures outlined in Chapter III. Tables, graphs, and diagrams are used to present the test results.

### 1-3 SCOPE

The investigation included four series of beams. The first series consisted of Beams V101-V107, the second, Beams V121-V127, the third, Beams V201-V207, and the fourth, Beams V221-V227. All specimens had a nominal cross section of 6 x 12 in. with an effective depth of 11 in. All beams, 10'-0 in length, were prestressed using high strength steel strand. Longitudinal and transverse reinforcement was provided for all beams.

The testing equipment allowed independent application of the twisting moment and the transverse load. Four beams were subjected to combined shear and flexure, and the remaining were tested under shear, flexure, and torsion. All were tested to failure by applying the loads



in a series of increments. The results of the tests are presented as MOMENT-DEFLECTION curves, TORQUE-TWIST curves, and DIMENSIONAL and NON-DIMENSIONAL INTERACTION DIAGRAMS, as well as in the form of tables and discussions.





## CHAPTER II

### REVIEW OF PREVIOUS RESEARCH

#### 2-1 INTRODUCTION

Although investigations dealing with torsion of reinforced concrete beams have been carried out over the years, studies concerned with the interaction of flexure, shear and torsion have only recently been performed. Tests concerned with prestressed concrete members have been relatively few. Initially, their behavior under torsion was studied, and later, investigations were made into the behavior of prestressed concrete members under bending, shear and torsion, as this is probably the most common loading condition found in practice. This chapter presents a review of previous research with emphasis given to tests dealing with members subjected to various loading combinations.

#### 2-2 PREVIOUS RESEARCH

Test of prestressed concrete sections under pure torsion have been carried out by various investigators. Humphreys tested 94 axially and eccentrically prestressed sections in pure torsion and found results could be predicted satisfactorily using the elastic theory. The specimens failed in a diagonal tension manner unless such failure was inhibited by high prestress.

Zia (2) conducted tests on 68 prestressed and plain concrete



members consisting of rectangular, 'I' and 'T' sections. Some of the specimens also contained web reinforcement and all were subjected to pure torsion. He reported abrupt failure for rectangular and 'T' sections, while the 'I' sections displayed considerable ductility. They refused to fail until the entire section including the flanges attained ultimate tensile strength. The specimens with web reinforcement showed considerable ductility. However, he stated that a concrete member without web reinforcement fails abruptly under torsion, and that this brittle type of failure can be avoided by providing web steel spaced not more than 0.4 of the depth of the member. He also decided that the use of transverse reinforcement has no apparent effect on the elastic torsional behavior of a member. He stated that the ultimate strength of prestressed members with web steel is equal to the sum of the cracking moment of the member and the contribution of the web reinforcement. He preferred the elastic theory for predicting the strength of specimens.

Mukherjee and Kemp (9) reported on tests conducted on plain, prestressed and reinforced concrete members subjected to pure torsion. The prestressed series included 18 beams without reinforcement and 15 beams with mild steel longitudinal and transverse reinforcement. The reinforced beams were 6 x 12 in. in cross section. They found that the addition of prestress caused a significant increase in ultimate torque in comparison to the ultimate capacity of a companion plain concrete beam. The inclination of the failure plane to the horizontal became progressively less with the increase in the intensity of prestress. This suggested the



principal tensile stress as the primary cause of failure. The prestressed beams exhibited greater plasticity than the plain concrete beams. The torsional failure of the prestressed concrete beams occurred suddenly and violently. Increased prestress was beneficial in enhancing the torsional capacity up to a limiting value beyond which the torsional capacity was reduced due to a compression failure of the concrete. The cracking torque of a prestressed beam with longitudinal and transverse reinforcement was comparable to that of an equivalent prestressed beam without reinforcement. Hence the effect of reinforcement could be conservatively ignored in estimating the cracking torque. They found that the range between the lower limit of cracking torque and the upper limit of compression failure torque was sharply reduced with an increase in the degree of prestress. This implies that at higher degrees of prestress, the ultimate torque can not be increased very much beyond the cracking torque, and that adequate ductility can not be ensured by providing mild steel reinforcement. Therefore, they decided that the degree of prestress should be limited to a maximum value.

Kemp, Sozen, and Siess (4) reported on a research project carried out to determine the behavior of concrete sections under torsional loading only. They found that the primary factors affecting the strength of prestressed members without web reinforcement were the shape of the cross section, the type and magnitude of the applied prestressing force, and the concrete strength. The various concrete sections behaved elastically up to cracking regardless of whether they were plain, or reinforced longitudinally or transversely. The stiffness and cracking torque appeared to





depend almost entirely on the geometry of the section and the concrete strength, and very little on the amount or position of any reinforcement provided. Upon reaching the cracking torque, the reinforced specimens continued to gain strength although losing stiffness until the ultimate torque was reached. This increase in strength was observed to depend primarily on the amount and location of the reinforcement present.

Nylander tested 60 specimens in various combinations of bending moment, torque and shear. While some of his specimens were plain, others were longitudinally reinforced. He preferred the plastic theory for predicting the ultimate capacity. He found that for low stress in the reinforcement, the bending moment exerted a favorable influence on the torsional strength of a beam. Since current codes insist on providing reinforced concrete beams with at least nominal transverse reinforcement, it is doubtful if his test specimens can be considered typical. He concluded that the ultimate capacity of a member subjected to bending, torsion and shear could be obtained by equating the sum of the torsional and transverse shearing stresses to the ultimate tensile strength of the concrete.

McMullen (3) tested 34 reinforced concrete beams; 22 of which were subjected to various combinations of bending and torsion, and 12 under various combinations of bending, torsion and shear. He concluded that reinforced concrete beams subjected to bending, torsion, and moderate amounts of transverse shear can fail by three different modes. These are



characterized by the formation of a hinge adjacent to one face of the beam and yielding of the reinforcement adjacent to the face opposite to the hinge. The modes of failure predicted by his analysis agreed with the observed modes of failure. He found that the presence of flexure does not increase the torsional strength of a beam provided with equal top and bottom reinforcement. He also found that the behavior of a beam prior to cracking is not significantly affected by the reinforcement provided. After cracking occurs, behavior depends on the reinforcement and on the ratio of twisting moment to bending moment.

Pandit and Warwaruk (7) reported results of tests performed on reinforced concrete sections subjected to combined bending and torsion. The results showed that the presence of flexure to a certain limit increases the torsional strength of a typical beam section in which most of the longitudinal steel is located in the zone of flexural tension. This increase in strength is essentially due to the greater resistance offered by the portion of concrete compressed by flexure. The presence of flexure reduces the torsional strength of a beam in which the longitudinal steel is distributed equally in the zones of flexural tension and compression, unless the ratio of transverse steel to longitudinal steel is low.

Swamy (14) reported tests conducted on 20 hollow rectangular prestressed concrete beams with comparison tests on prestressed solid and plain hollow concrete beams. The specimens were tested under varying ratios of bending moment to twisting moment. He stated that the torsional





resistance of a hollow beam can be increased by the presence of bending moment. He also concluded that the mode of failure is dependent on the ratio of torque to bending. He found that the presence of the prestress tends to retard the development of failure strains, thereby increasing the capacity of the section in torsion. He concluded that interaction curves were the best method for assessing the capacity under various stress combinations ranging from pure bending to pure torsion.

Cowan and Armstrong (1) conducted tests to determine the behavior of reinforced and prestressed concrete beams subjected to combined bending and torsion. They concluded that the crack patterns depended upon the ratio of bending to torsional moment. Tests under pure bending produced vertical cracks while specimens under pure torsion exhibited cracking along lines  $45^\circ$  to the horizontal axis of the beam. Specimens under combined loading exhibited crack patterns within these two limits. Also, the addition of prestress flattened the crack pattern to possible angles less than  $45^\circ$  depending upon the level of prestress. A sizeable reduction in stiffness was noted after cracking but the specimens possessed additional capacity over and above their cracking capacity. This increase was attributed to the reinforcement and depended not only on the amount but on its location within the section. Transverse reinforcement was not included in their specimens and was therefore not discussed. They found that the presence of a bending moment, less than the flexural cracking moment, was beneficial to the torsional capacity of a specimen, and the presence of a torsional moment, less than the torsional cracking



moment, was beneficial to the flexural capacity of the specimen. Cowan suggested that the torsional strength of a beam was equal to the sum of the torsional strength of the plain concrete and the contribution of the reinforcement. He advocated the elastic theory approach, and concluded that closer agreement between experimental results and theory would be obtained if the torsional strength of the concrete was considered and the reinforcement required to take only those tensile stresses which exceeded the tensile strength of the concrete.



## CHAPTER III

### TEST SPECIMENS, INSTRUMENTATION, EQUIPMENT AND PROCEDURE

#### 3-1 TEST SPECIMENS

The twenty four beams reported in this study were all provided with both longitudinal and transverse reinforcement. They were divided into four groups as outlined in TABLE 3.3. One beam from each series was tested under shear and bending; the remaining specimens were subjected to torsion, shear and bending. All beams had a nominal cross section of 6 x 12 in. and their overall length was 10'-0.

##### (a) Concrete

The concrete mix was the same for all beams and was comprised of the following proportions:

(1) CEMENT (TYPE III)	133 lbs.
(2) SAND	344 lbs.
(3) COARSE AGGREGATE	500 lbs.

The amount of water used per mix was approximately 85 lbs. This mix yielded seven cubic feet of concrete with a 3 inch slump.

##### (b) Sand

A sieve analysis of the sand is given in TABLE 3.1. The average moisture content was approximately 4%.





SIEVE SIZE	WEIGHT RETAINED (gms.)	% RETAINED	CUMULATIVE % RETAINED	A.S.T.M. STANDARD
# 4	17.5	3.0	3.0	0 - 5
# 8	85.2	14.7	17.7	
# 16	54.6	9.5	27.2	20 - 55
# 30	60.0	10.3	37.5	
# 50	208.4	35.8	73.3	70 - 90
#100	122.9	21.1	94.4	90 - 98
PAN	17.8	3.1	-	
SILT	14.4	2.5	-	
TOTAL	580.8	100.0	253.1	
FINENESS MODULUS 2.53				

TABLE 3.1 SIEVE ANALYSIS OF SAND

SIEVE SIZE	WEIGHT RETAINED (lbs.)	% RETAINED	CUMULATIVE % RETAINED
3/4"	0.30	1.1	1.1
3/8"	15.63	58.4	59.5
# 4	10.03	37.5	97.0
PAN	0.80	3.0	100.0
TOTAL	26.76	100.0	

TABLE 3.2 SIEVE ANALYSIS OF COARSE AGGREGATE



BEAM NO	LONGITUDINAL REINFORCEMENT		TRANSVERSE		REINFORCEMENT		CONCRETE STRENGTH		PRESTRESS	
	DESCRIPTION	P <sub>1</sub> %	f <sub>y1</sub> ksi	DESCRIPTION	P <sub>t</sub> %	f <sub>yt</sub> ksi	f' <sub>a</sub> psi	f' <sub>sp</sub> psi	TOTAL EFFECTIVE FORCE	$\frac{e}{d}$
101	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	3685	281	36.29	0
102	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	3988	325	37.78	0
103	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	4280	424	37.65	0
104	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	4846	396	33.81	0
105	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	4118	389	33.31	0
107	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	3685	332	36.04	0
121	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	4236	460	35.08	0.167
122	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	4268	433	32.34	0.167
123	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	4263	411	32.23	0.167
124	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	4392	469	35.12	0.167
125	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	4870	484	35.65	0.167
127	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	4619	462	34.12	0.167
201	4 # 3	0.61	56.4	#2 @ 3	0.70	55.5	5158	513	82.39	0
202	4 # 3	0.61	56.4	#2 @ 3	0.70	55.5	5000	438	78.57	0
203	4 # 3	0.61	56.4	#2 @ 3	0.70	55.5	4914	484	77.03	0
204	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	5701	500	85.37	0
205	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	5618	497	84.81	0
207	4 # 3	0.61	56.4	#2 @ 3	0.70	55.5	5041	543	81.53	0
221	4 # 3	0.61	56.4	#2 @ 3	0.70	55.5	5353	442	79.33	0.167
222	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	5766	538	80.60	0.167
223	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	5813	464	80.12	0.167
224	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	5931	513	80.25	0.167
225	4 # 3	0.61	56.4	#2 @ 3	0.70	40.3	5389	432	80.04	0.167
227	4 # 3	0.61	56.4	#2 @ 3	0.70	55.5	5064	475	79.19	0.167

TABLE 3.3 PROPERTIES OF TEST SPECIMENS





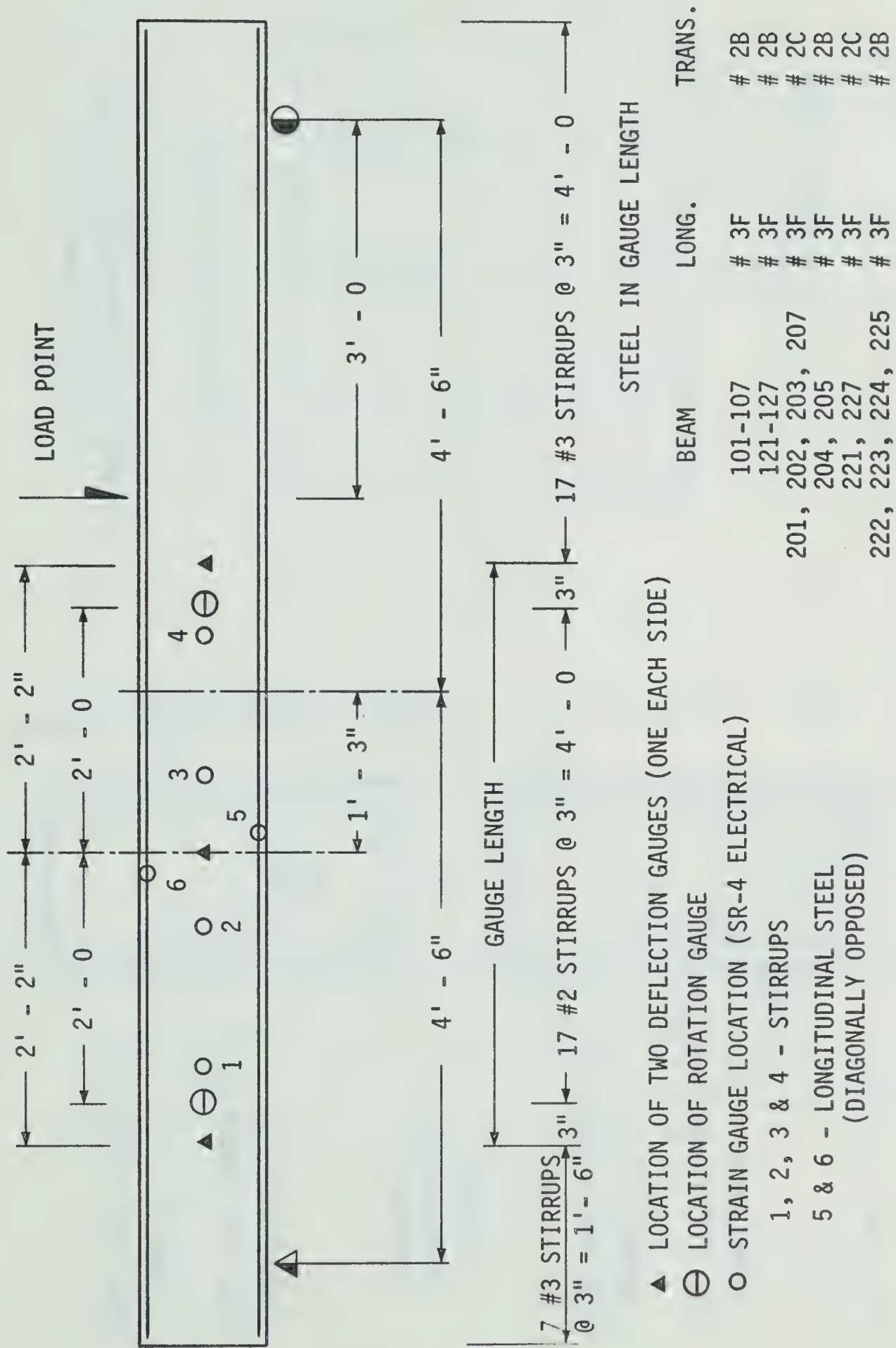


FIGURE 3.1 TYPICAL SPECIMEN VIEWED FROM NORTH SIDE



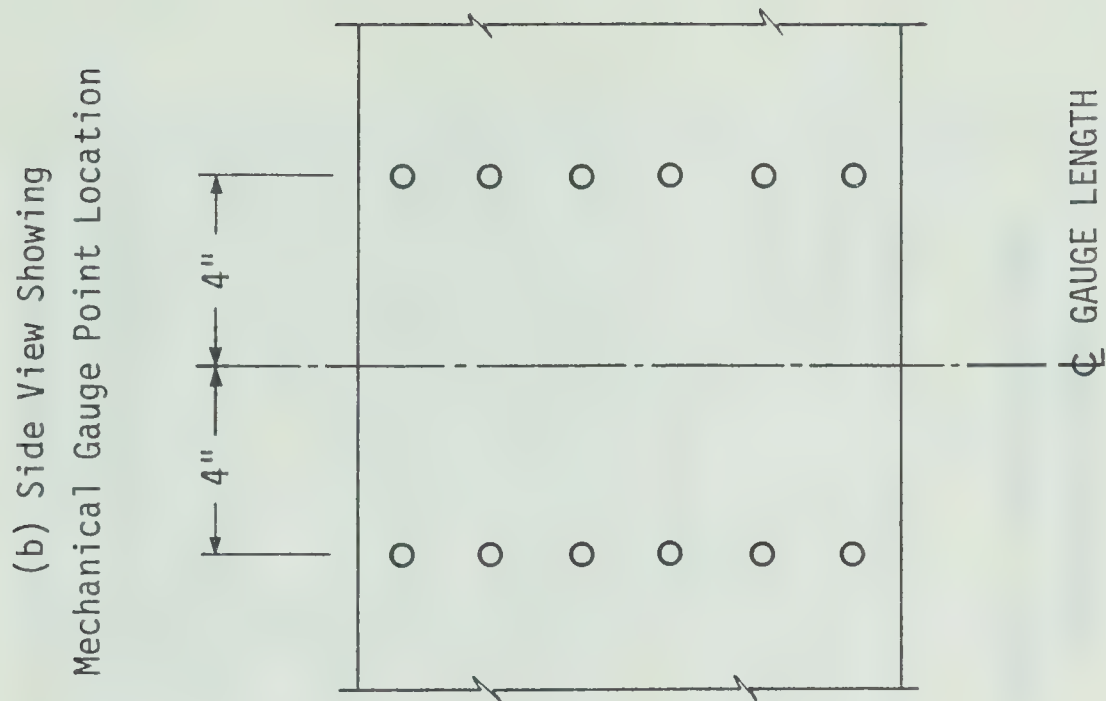
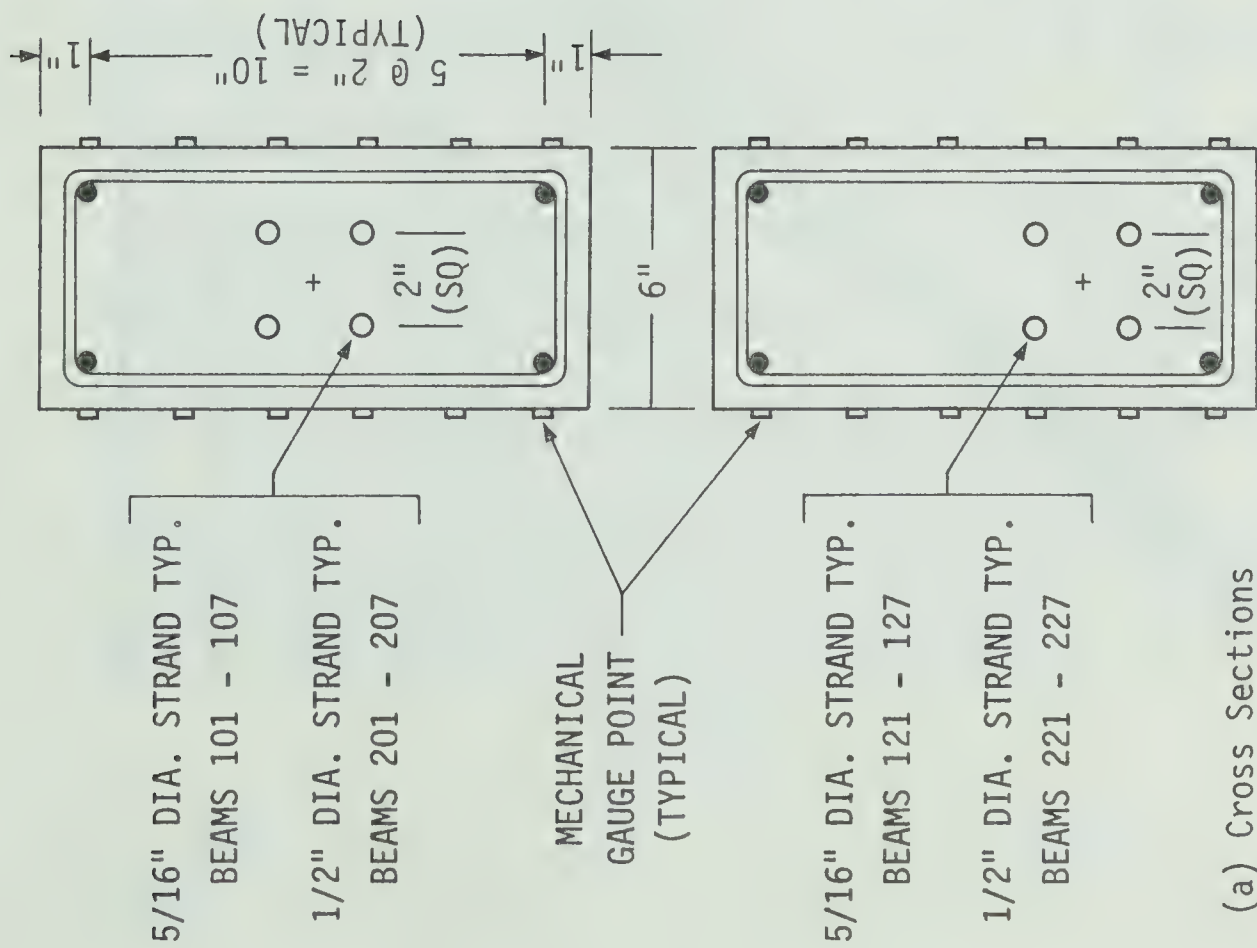
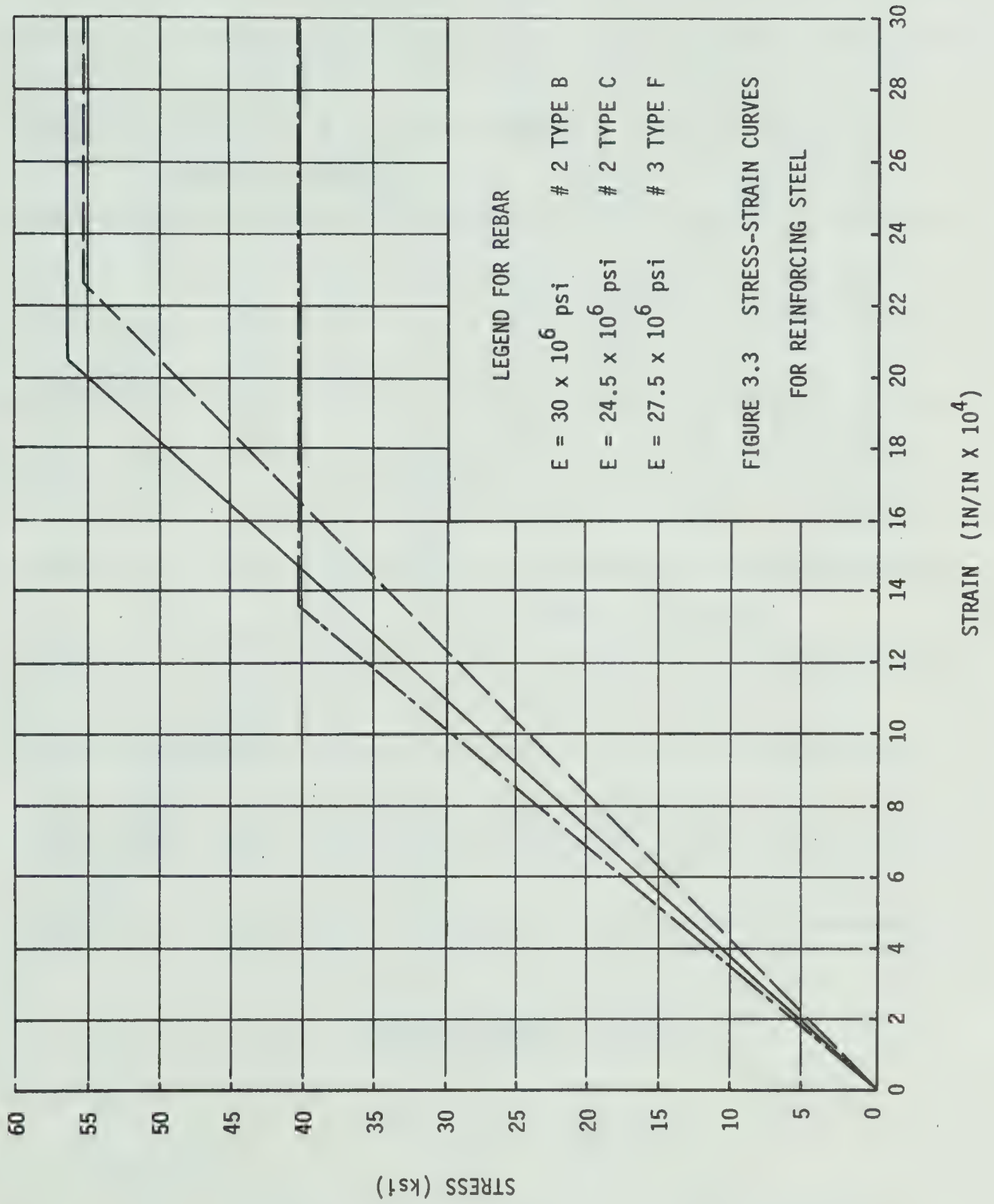


FIGURE 3.2 BEAM SECTIONS (V 101 - V 227)









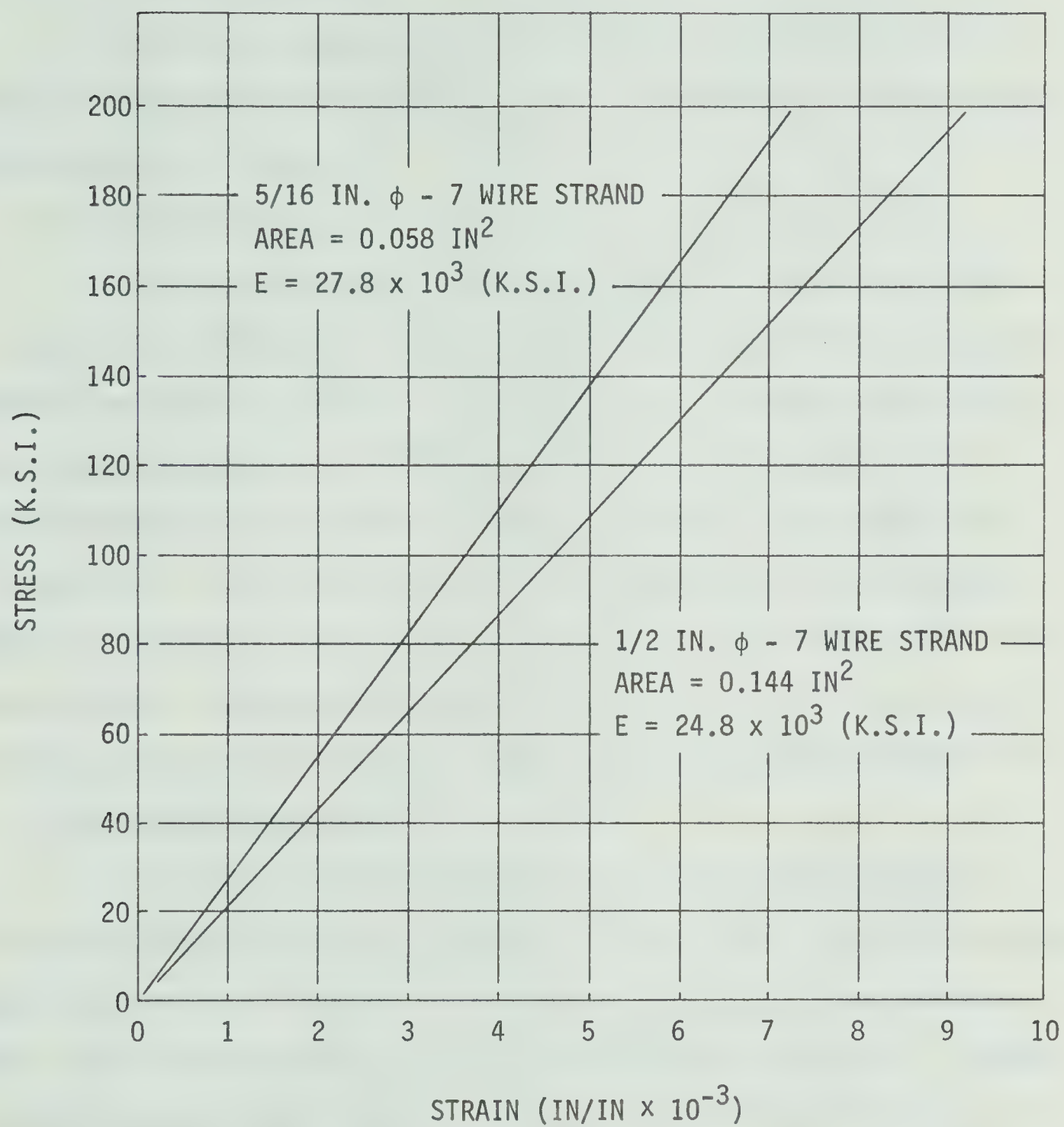


FIGURE 3.4 STRESS-STRAIN CURVES FOR PRESTRESSING STRAND



### (c) Coarse Aggregate

The coarse aggregate was 3/4 in. maximum size crushed rock with an average moisture content of 1.7%. The results of a sieve analysis are presented in TABLE 3.2.

### (d) Reinforcement

The non-prestressed reinforcement used in the test specimens is described in TABLE 3.3. The #3 deformed bars were from the same heat and are designated as TYPE F. The #2 plain bars were from two different heats and are designated as TYPE B or TYPE C. The arrangement of the reinforcement in the specimens is shown in FIGURE 3.1 and FIGURE 3.2a. To ensure that failure would occur in the gauge length, a considerable amount of transverse reinforcement was provided in the areas outside the gauge length. Representative samples of the #3 bar and of each type of the #2 smooth bar were tested to obtain the results shown in FIGURE 3.3.

### (e) Prestressing Strand

For test specimens V101-V107 and V121-127, the strand used for prestressing was 5/16 in. diameter - 7 wire strand. For test specimens V201-V207 and V221-V227, 1/2 in. diameter - 7 wire strand was employed. Both sizes were guaranteed a minimum yield strength of 250 ksi. A representative strand sample from each size was tested to obtain the results shown in FIGURE 3.4.

### (f) Fabrication

The reinforcement cages were fabricated by fastening the stirrups to the longitudinal reinforcement with ties at 3" intervals. In preparation for casting the steel forms were cleaned and oiled. The reinforcement





cages having been placed into position, the prestressing strands were threaded into place and the side forms were set alongside the cages and bolted securely into place. The strand length was such that it extended beyond each bulkhead four to five feet. At the north bulkhead, load cells and wedge-grip end anchorages were installed. FIGURE 3.5 illustrates a typical arrangement. The south bulkhead shown in FIGURE 3.6 was used as the stressing point and the wedge-grip end anchorages were again used to hold the elongation in the strand upon prestressing.

The entire system was properly aligned and the strands were individually stressed using a Simplex centre-hole hydraulic jack operated by an electric pump. It was attempted to stress each strand to the same level of prestress but variations in end anchorage losses caused small variations to occur in any one particular beam.

The concrete was mixed in a nine foot capacity mixer located within the laboratory. One batch was sufficient for each beam including its control cylinders. Prior to mixing, a butter mix was used to condition the mixer. The concrete was mixed for approximately five minutes with the water content adjusted until a 3" slump was obtained. The concrete was then transported to the forms and cast into place with the aid of an internal vibrator.

Five six by twelve inch control cylinders were cast with each specimen and were cured and stored under identical conditions. Three cylinders were used for compression tests and the remaining two for splitting tensile tests. All the control cylinders were tested on the same day as the corresponding beam test.





FIGURE 3.5 NORTH BULKHEAD

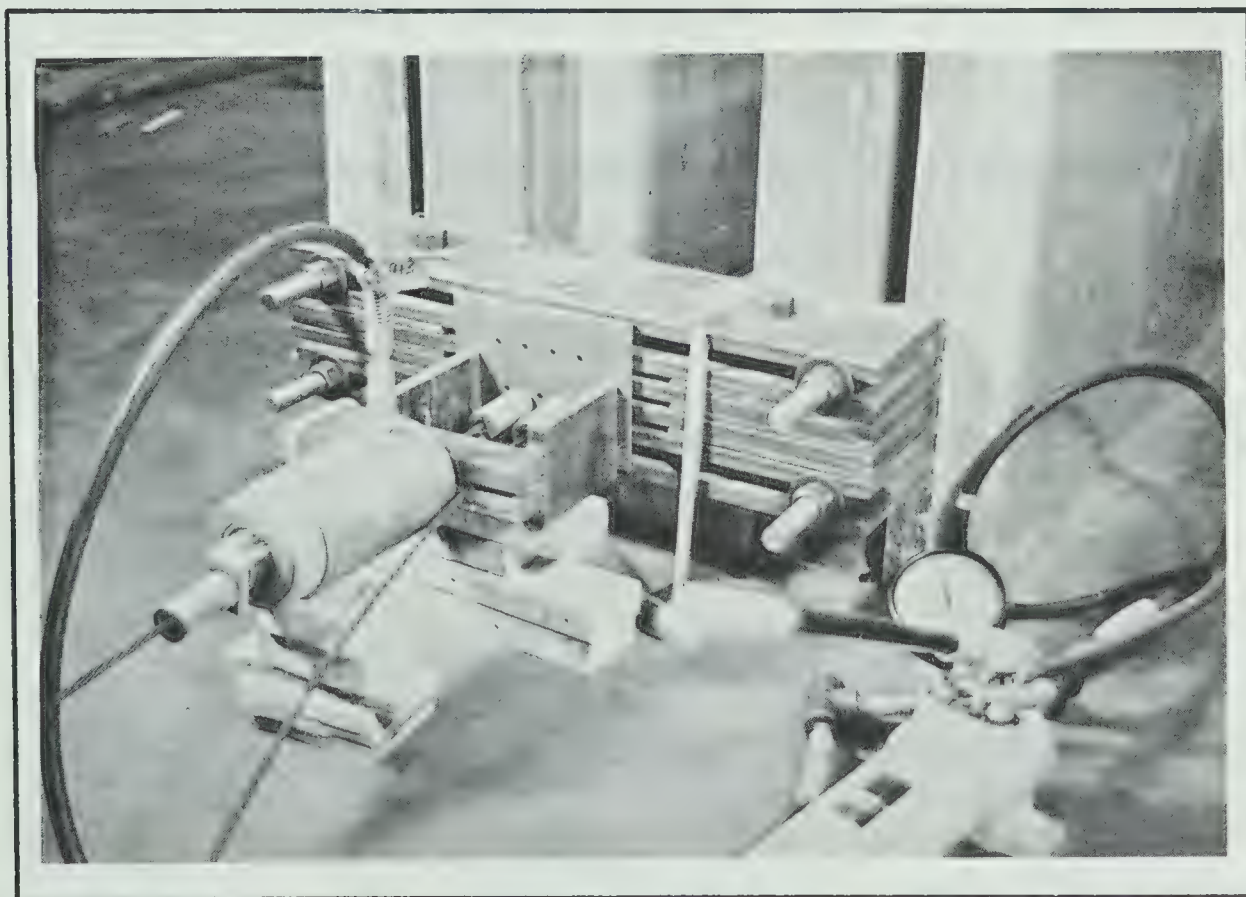


FIGURE 3.6 SOUTH BULKHEAD  
(JACKING SYSTEM)





The day after casting the forms were removed and the specimens and their cylinders were covered with moist burlap and plastic for six days.

At seven days the plastic and burlap were removed and final readings were taken on each load cell to eliminate any error in strand forces due to relaxation of the steel. Mechanical gauge points were positioned on both vertical faces and initial readings were taken using an 8 inch DEMEC deformation gauge. FIGURE 3.1 and FIGURE 3.2 illustrate the location of these gauge points. The strands were then subjected to heat from a cutting torch over an approximate length of two feet such that the stress was allowed to gradually transfer from the strands to the concrete.

After release a second reading was taken on all the gauge points for purposes of calculating the 'instantaneous' elastic shortening in the concrete due to prestress. The beams and their cylinders were then set aside to cure in air until the time of their test.

### 3-2 INSTRUMENTATION

The specimens were instrumented to enable measurement of angle of twist, deflections and reinforcement strains.

#### (a) Angle of Twist

The twistmeters were designed to measure the total angle of twist and their location on each specimen is shown in FIGURE 3.1. Each consisted of a level bubble mounted on a 1 x 1-1/2 in. channel and fixed at one end by a pin joint. The opposite end was supported by the needle





of a micrometer screw mounted on the base. Small springs on either side of the channel ensured its close contact with the micrometer screw needle. A clamping bracket attached this assembly to the top face of the beam. The smallest division on the micrometer was 0.001 in. The angle of twist through which each twistmeter rotated was computed from the difference in the micrometer readings between successive load increments. The total angle of twist over the gauge length was calculated using the difference between the angles of twist obtained from each twistmeter.

#### (b) Deflections

The beam deflections were measured at three locations which are shown in FIGURE 3.1. The deflection gauges consisted of scales suspended from rods at equal distances from the beam face. These rods which projected horizontally on either side of the beam at mid depth were fastened to a bracket clamped onto the beam. The smallest division on these scales was 0.01 in. Readings were taken using two precise levels located on either side of the specimen; the deflection was obtained by averaging each corresponding pair of readings. The deflection evaluated in this manner was the vertical deflection of the centre of the cross section due to the applied transverse load.

#### (c) Reinforcement Strains

At strain gauge locations the bars were ground smooth and Type A-7 SR-4 electrical resistance strain gauges were attached. The locations of these strain gauges were identical for all beams and are shown in FIGURE 3.1. After the gauges were cemented in place, they were



waterproofed with three coats of GW-2 waterproofing compound and wrapped with tape. The surrounding area was then treated with an epoxilite compound to protect the strain gauges during casting.

### 3-3 TEST EQUIPMENT

The arrangement used for testing the beams is illustrated in FIGURE 3.10. The apparatus employed in applying the torsional moment to the specimen was completely independent of that used to apply the transverse shear and bending moment. The transverse load was applied by means of a 100 kip Amsler jack which rested on a heavy plate equipped with rollers. This plate was laterally supported and rested on a pipe collar which was fastened to the specimen at the point of loading. This system not only permitted the twisting moment to be transmitted fully along the length of the beam but also ensured that the transverse load remained in a vertical orientation.

The east end of each specimen was supported by the twisting head through which the torsional moment was applied. FIGURE 3.7 illustrates the twisting head. It allowed the beam to rotate about its longitudinal axis and also about both a horizontal and a vertical axis perpendicular to the axis of the specimen. Cables were attached to the arms of the twisting head and previously calibrated load cells were used to measure the forces in these cables.

The cable forces were produced by the torsional loading system shown in FIGURE 3.8. The cables connected the torsion arms of the twisting head to the ends of the cross head. A roller assembly provided at each



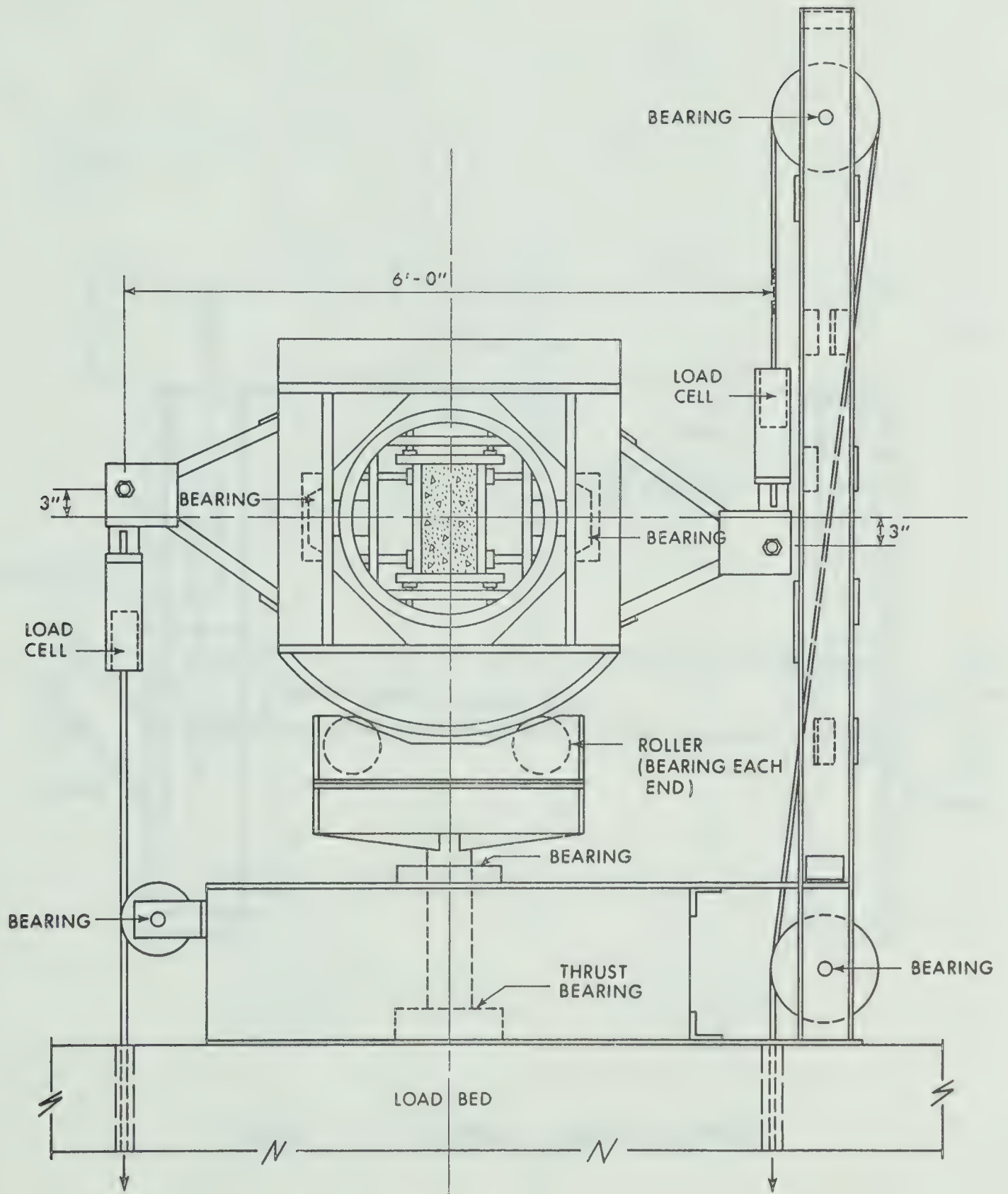


FIGURE 3.7 TWISTING HEAD





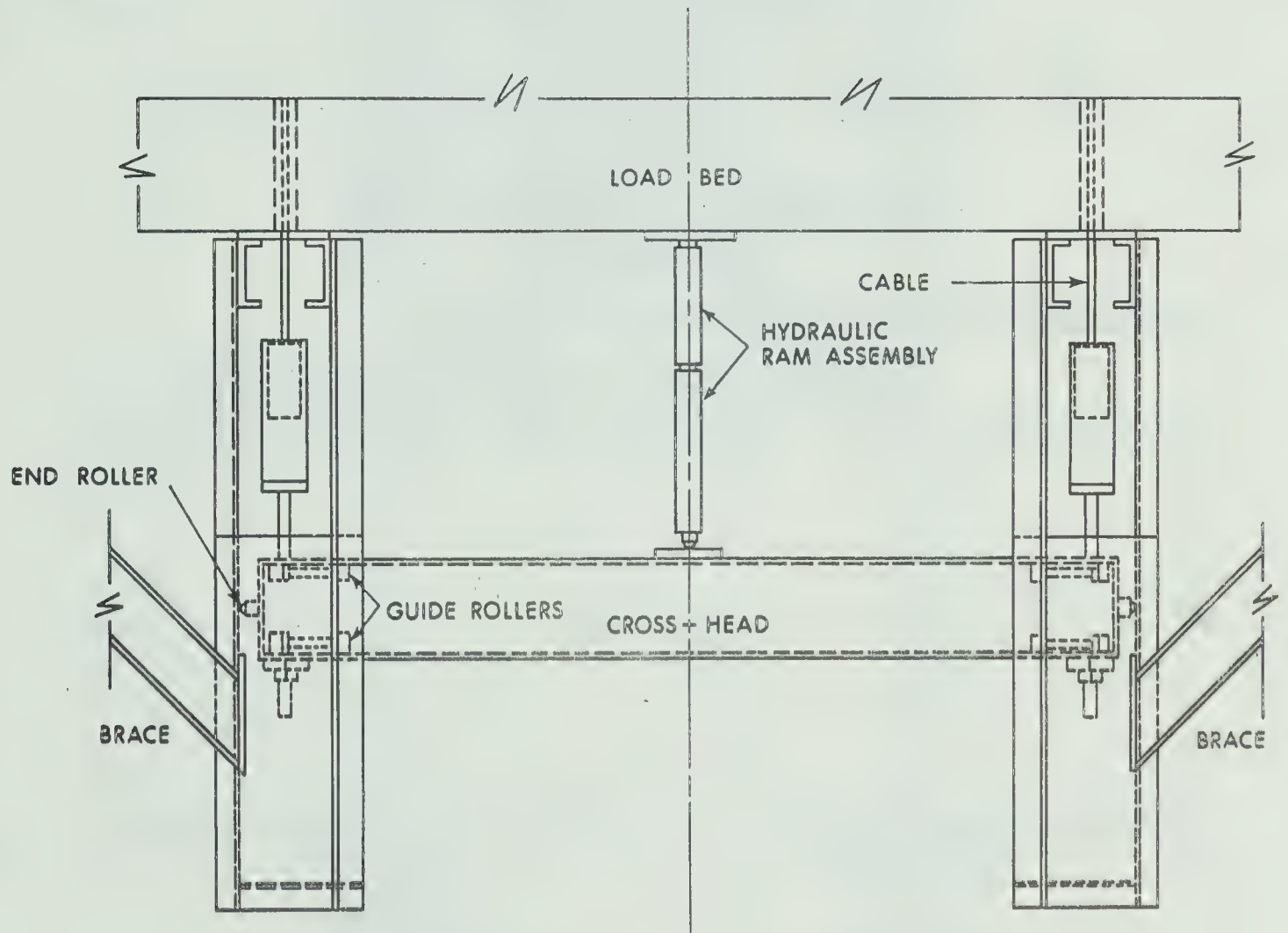


FIGURE 3.8 TORSIONAL LOADING EQUIPMENT



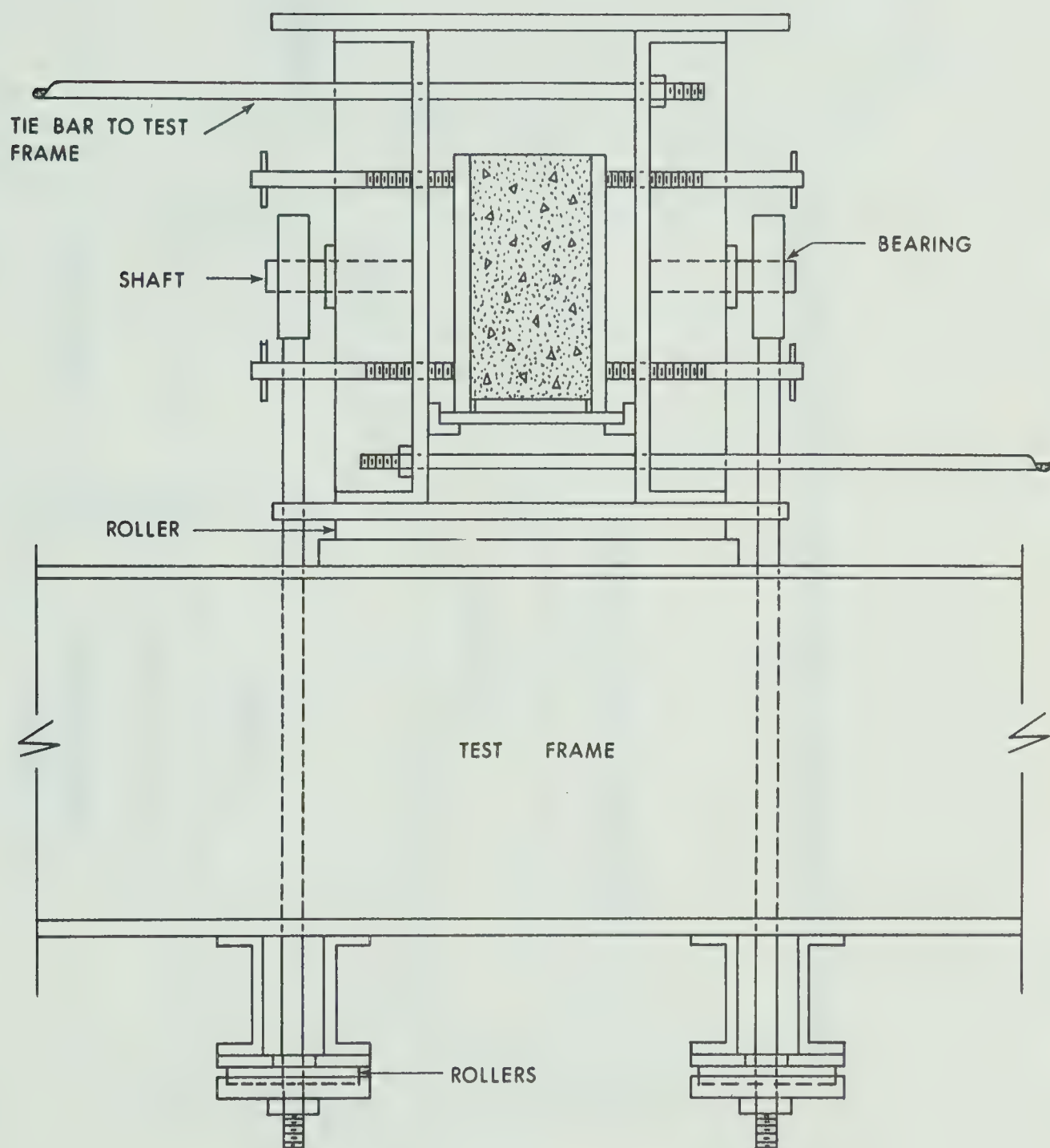


FIGURE 3.9 FIXED HEAD



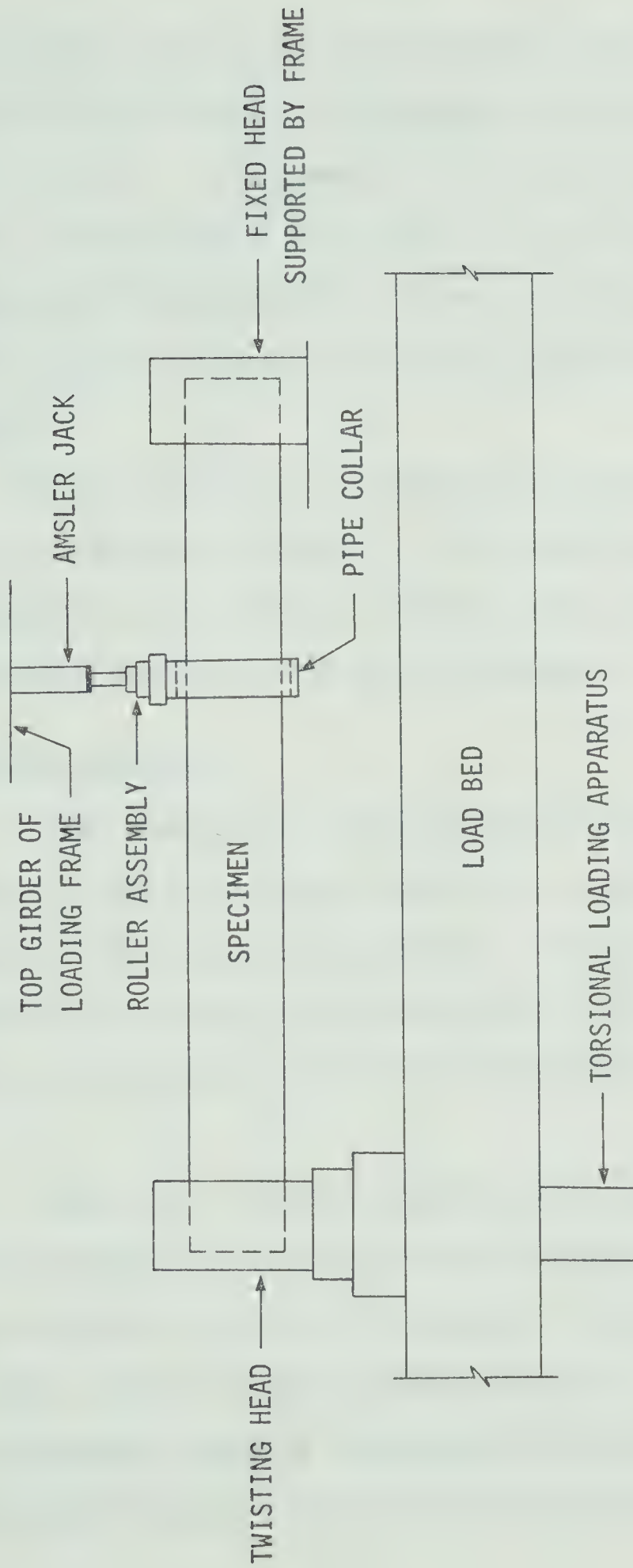


FIGURE 3.10 EQUIPMENT ARRANGEMENT VIEWED FROM NORTH SIDE





end of the cross head ensured free movement within the guides provided. The cross head was loaded at its midspan by two 20 kip hydraulic jacks coupled in series. Use was made of a hand pump to generate the hydraulic pressure. The coupling of the jacks in series was intended to increase the stroke to 16" corresponding to a rotation of approximately  $26^\circ$  in the twisting head, slightly more than the maximum rotation of which it is capable.

The west end of the specimen was supported by the fixed head which is illustrated in FIGURE 3.9. This support permitted translation in a longitudinal direction and rotation about a horizontal axis perpendicular to the longitudinal axis of the beam.

### 3-4 TESTING PROCEDURE

Final readings were taken on the mechanical gauges so as to calculate the loss in prestress force due to time dependent strains which had occurred from release up to testing. The specimen was then placed into position and secured. The dead weight of the pipe collar and roller assembly had been determined beforehand and allowance was made for their effect.

Beams V101, V121, V201 and V221 were tested under combined shear and flexure without the presence of any torsional moment. The transverse load was applied in a series of increments. The remaining beams were tested under combined torsion, bending and shear. The twisting moment and the transverse load were incremented simultaneously in predetermined magnitudes depending upon the ratio of torsional moment to bending moment



for each particular specimen. When a specimen reached a critical stage in the test such as cracking or ultimate load, the loading increments were reduced such that more data could be gained in these regions. Ultimately, each specimen was tested to failure and note was made of the approximate distance of the failure region from the point of loading. For each loading increment, all the instrumentation was read, and the crack pattern was marked.



## CHAPTER IV

### PRESENTATION OF TEST RESULTS

#### 4-1 INTRODUCTION

The following sections are included to related the manner in which the test results were obtained. The readings which were taken for each beam at the end of successive loading stages are included in APPENDIX A. The TORQUE-TWIST curves and the MOMENT-DEFLECTION curves are also presented in that section. The INTERACTION DIAGRAMS at the ultimate loading condition are included in the next chapter to be discussed at that point. Photographs of most of the test specimens after failure are presented in APPENDIX B.

Due to an oversight in the preparation of Beam V103 for testing, the results obtained were considered possibly erroneous, and as a result, no photographs of this specimen after failure were taken. Also, during the testing of Beam V222, a power failure necessitated releasing the load and reloading again at a later date, and the results obtained beyond the point of initial loading could not be considered typical.

#### 4-2 TORQUE-TWIST RELATIONSHIPS

The torsional moment acting upon the specimen was evaluated using the product of the load cell reading and the moment arm of 72 in. At each load increment, the twist was calculated using successive readings





obtained from the two rotation gauges. Subtraction of the west gauge readings from those of the east gauge resulted in the angle change between them. Dividing this angle change by the distance between gauges yielded the angle of twist in radians per inch.

The values are plotted in the form of TORQUE-TWIST curves and these are shown in APPENDIX A.

#### 4-3 MOMENT-DEFLECTION DIAGRAMS

The location of the deflection gauges is shown in FIGURE 3.1. Readings taken at each loading increment were averaged for each pair of gauges; the average result being the downward deflection of the centroid of the specimen due to the applied transverse load.

The maximum flexural moment along the length of the test specimen occurred under the transverse load. For each load sequence, it was evaluated as one-third the value of the transverse load multiplied by 72 in. to give a value in units of inch kips. The value of the bending moment at any other section along the specimen could be easily obtained from statics.

The main objective in plotting moment-deflection diagrams was to show the effect of three variables upon the behavior of the beam; the degree of prestress, the position of the prestress, and the ratio of the torsional moment to bending moment. To show the effect of both the degree and the location of the prestress force, FIGURE A.5 was drawn using as coordinates the maximum flexural moment under the load and the deflection of the west gauge for those beams tested without the presence of torsional



moment. Then, using the same coordinates, moment-deflection diagrams for each series of beams were plotted to illustrate the effect of varying the ratio of torque to bending for different degrees and locations of prestress. This is shown in FIGURE A.6 and FIGURE A.7. For more complete information concerning each beam, reference can be made to the tables in APPENDIX A.

#### 4-4 VALUE OF THE EFFECTIVE PRESTRESS FORCE

TABLE 3.3 shows the value of the effective prestress force for each beam. Measurements were taken to determine the elastic shortening and the time dependent losses as previously outlined in Section 3-4. Modification of these strain losses to stresses and subsequently to forces resulted in determination of the total loss in the strand force occurring from time of release to time of testing. These losses were deducted from the strand force prior to release; the result being the effective prestress force at the time of testing.

#### 4-5 INTERACTION DIAGRAMS

When dealing with varying loading combinations, it has been a frequent practice to plot interaction diagrams with the various types of loading as coordinates. The effect that one type of loading has upon another can be determined in this manner. The study of two types of loading, namely bending and torsion, is relatively straightforward compared to the study of bending and torsion combined with transverse shear. The former can be illustrated on a two-dimensional plot while the combi-



nation of bending, shear and torsion is more aptly presented using a three-dimensional diagram.

For this study, interaction diagrams at ultimate load were plotted in two dimensions with the transverse shear and flexural moment individually plotted opposite the torsional moment. The non-dimensional plots are illustrated opposite the results of tests under combined bending and torsion performed by Sorensen (12) and Mukherjee (15). They are illustrated in FIGURE 5.1 and FIGURE 5.2. The plots showing the interaction of torsion and shear are shown in FIGURE 5.3. The value of the torsional moment at ultimate load,  $T_u$ , was simply that value of the twisting moment existing at failure. The value of the bending moment at ultimate load,  $M_u$ , was taken to be the moment existing at the failure plane section at the time of failure. After failure, the distance from the load point to the failure plane was recorded and is tabulated in TABLE 5.1. Because nearly all the failures took place over a finite length, the exact position of the failure plane was often an approximation, particularly in the cases of high torque to bending ratios. In these tests, failure occurred along a plane inclined to the horizontal and it was difficult to pinpoint an exact position for the failure plane. The value of the transverse shear along the gauge length at ultimate loading,  $V_u$ , was taken as one-third the value of the transverse load at failure. The values of  $M_{u0}$  and  $T_{u0}$  were taken from previous tests of similar beams subjected to pure bending and pure torsion respectively. Slight variation in effective prestress force and concrete strength between these specimens





and those of this present study were present, but it was felt that, despite these differences, the ratios obtained were quite acceptable. The value of  $V_{u0}$  was taken as the transverse shear along the gauge length at ultimate conditions for a beam subjected to shear and bending only.

Dimensional interaction diagrams were also plotted using transverse shear and flexural moment as abscissae and torsional moment as the ordinate. These are shown in FIGURE 5.4. The values used in plotting all the interaction diagrams are tabulated in TABLE 5.1.



## CHAPTER V

### DISCUSSION OF TEST RESULTS

#### 5-1 INTRODUCTORY REMARKS

The general behavior of the test beams over the loading sequence is discussed in this chapter with reference being made to the torque-twist curves and the moment-deflection curves illustrated in APPENDIX A. The comparison at ultimate between these test results and those obtained previously under combined bending and torsion are presented and discussed. In addition, the topics discussed in the following sections include the degree and location of the prestress, the ratio of applied torque to bending moment, and the effect of transverse shear on the behavior of prestressed beams subjected to combined loading.

#### 5-2 GENERAL BEAM BEHAVIOR AND CRACK PATTERNS

The shape of the torque-twist curves and the moment-deflection curves indicates the presence of two stages in the behavior of the test specimens under load. Initially until cracking, the beams behaved essentially elastically. However, after cracks had developed, it appeared that any further strength possessed by the specimen could be attributed to the presence of the longitudinal and transverse reinforcement. Examination of the strain gauge readings included in the beam tables in APPENDIX A reveals the contribution made by the reinforcement in extending the



BEAM NO	$\frac{\text{TORSIONAL MOMENT}}{\text{BENDING MOMENT}}$	ULTIMATE SHEAR IN GAUGE LENGTH (KIP)	ULTIMATE TORQUE (IN.KIP)	DISTANCE FROM FAILURE PLANE TO LOAD POINT	ULTIMATE MOMENT (IN.KIP)	$\frac{V_u}{V_{uo}}$	$\frac{T_u}{T_{uo}}$	$\frac{M_u}{M_{uo}}$
101	0	6.23	0	12" EAST	374	1.00	0	0.86
102	1/3	5.79	76	12" EAST	348	0.93	0.54	0.80
103	3/4	3.73	107	-	-	0.60	0.76	-
104	4/3	2.47	128	24" EAST	118	0.40	0.91	0.27
105	3	1.10	129	45" EAST	30	0.18	0.91	0.07
107	1/7	6.33	35	12" EAST	380	1.02	0.25	0.87
121	0	8.50	0	7" EAST	553	1.00	0	0.97
122	1/3	7.33	95	12" EAST	440	0.86	0.65	0.78
123	3/4	4.33	127	21" EAST	221	0.51	0.86	0.39
124	4/3	2.67	137	22" EAST	132	0.31	0.93	0.23
125	3	1.20	137	41" EAST	37	0.14	0.93	0.07
127	1/7	8.67	48	8" EAST	555	1.02	0.33	0.98
201	0	9.82	0	10" EAST	609	1.00	0	0.96
202	1/3	8.50	111	9" EAST	536	0.87	0.76	0.84
203	3/4	5.07	148	48" EAST	122	0.52	1.01	0.19
204	4/3	3.00	154	48" EAST	72	0.31	1.05	0.11
205	3	1.20	140	52" EAST	24	0.12	0.96	0.04
207	1/7	9.92	56	12" EAST	595	1.01	0.38	0.94
221	0	14.17	0	8" EAST	907	1.00	0	0.99
222	1/3	10.67	137	7" WEST	619	0.75	0.90	0.68
223	3/4	5.10	148	28" EAST	224	0.36	0.97	0.25
224	4/3	3.10	160	41" EAST	96	0.22	1.05	0.11
225	3	1.23	142	49" EAST	28	0.09	0.93	0.03
227	1/7	13.67	76	12" EAST	820	0.96	0.50	0.90

TABLE 5.1 DATA FOR INTERACTION DIAGRAMS





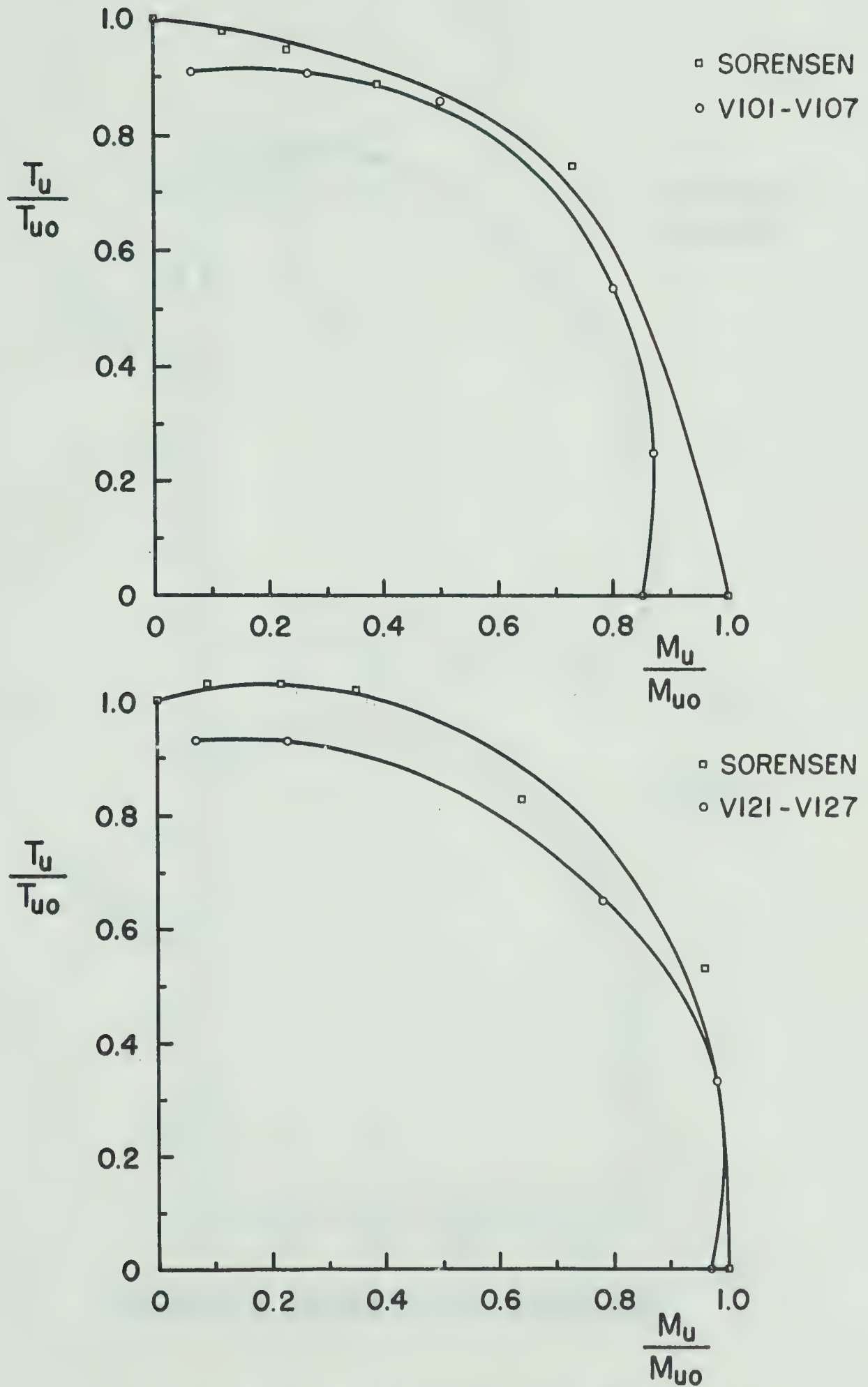


FIGURE 5.1 INTERACTION DIAGRAMS



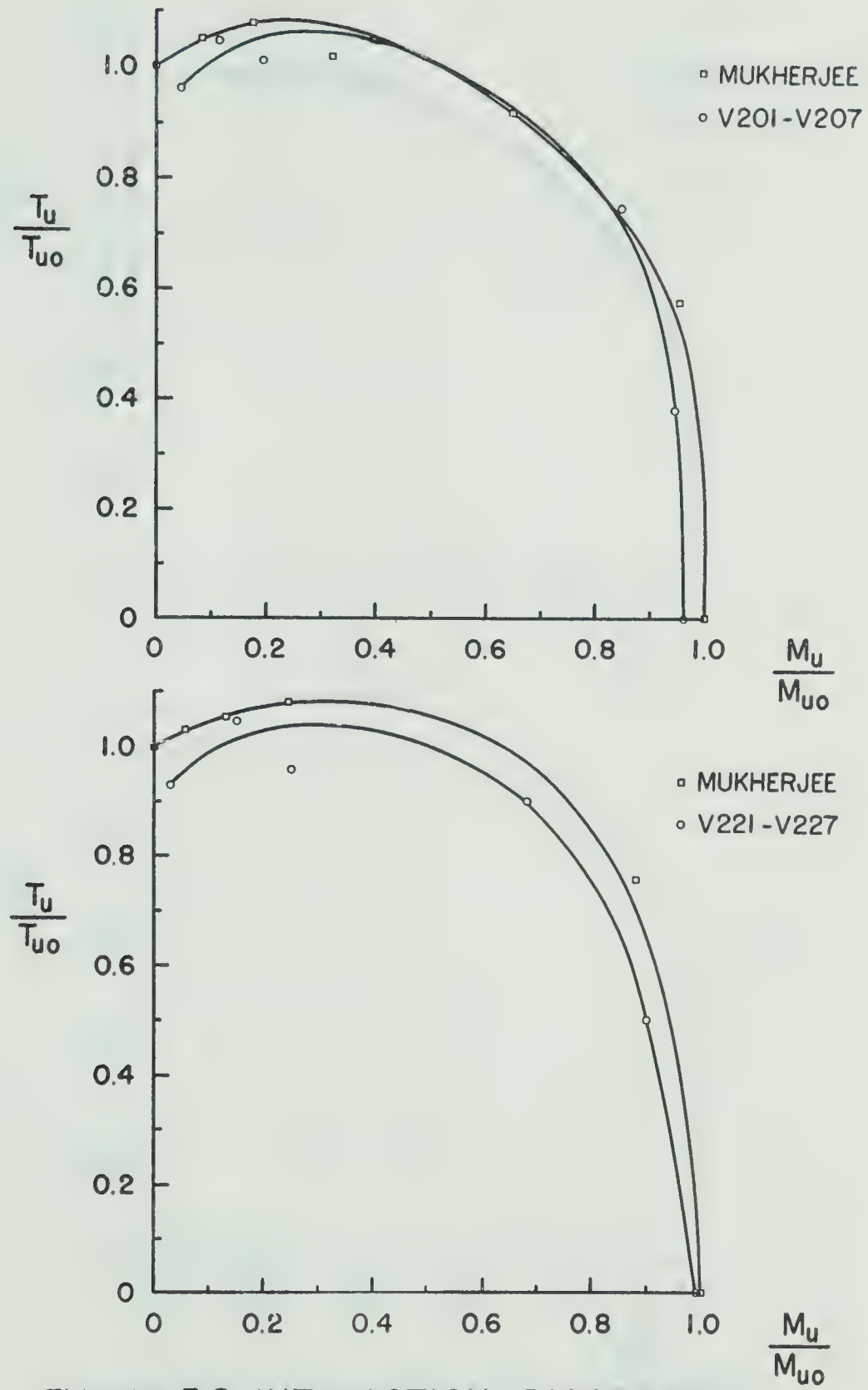


FIGURE 5.2 INTERACTION DIAGRAMS



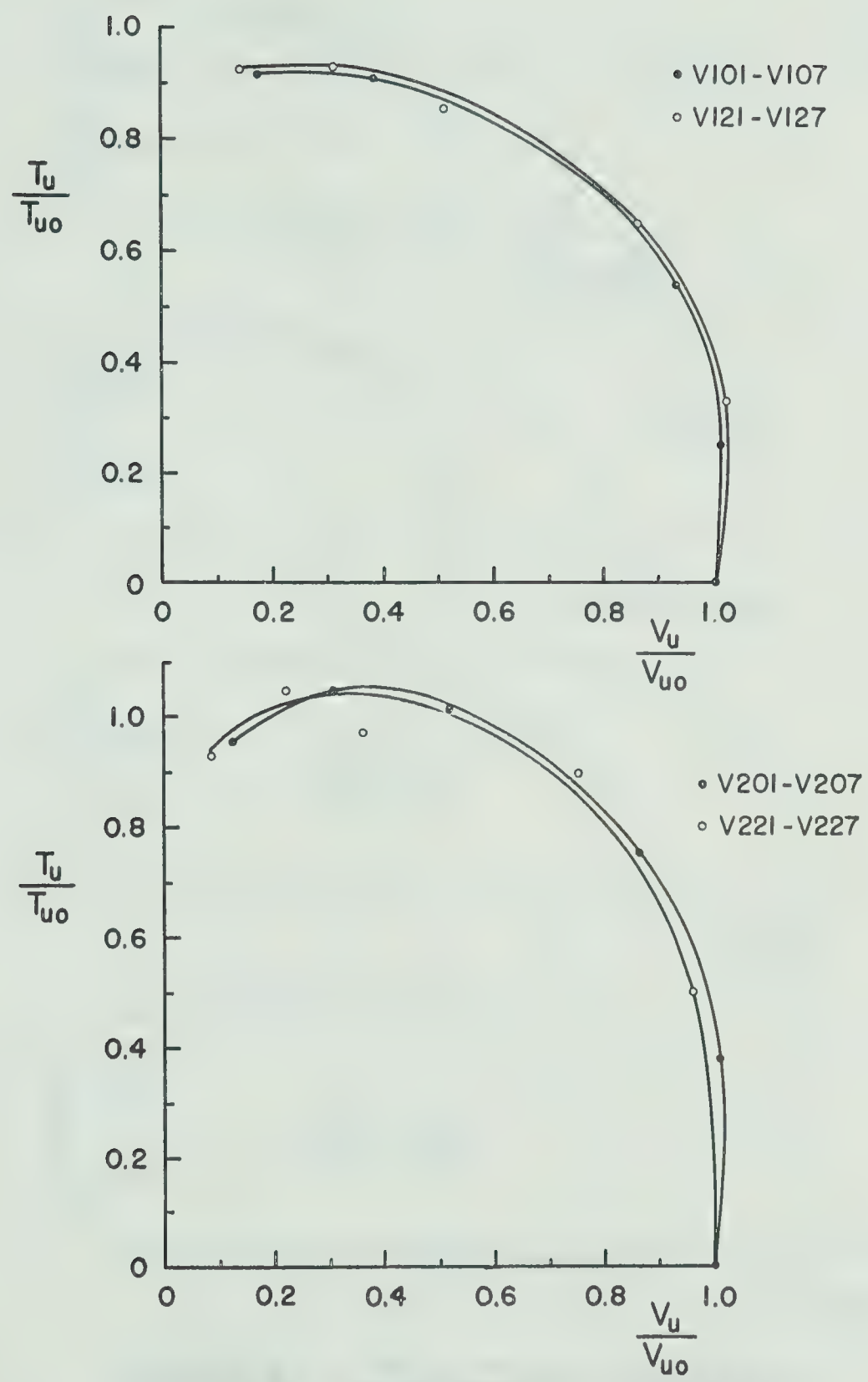


FIGURE 5.3 INTERACTION DIAGRAMS





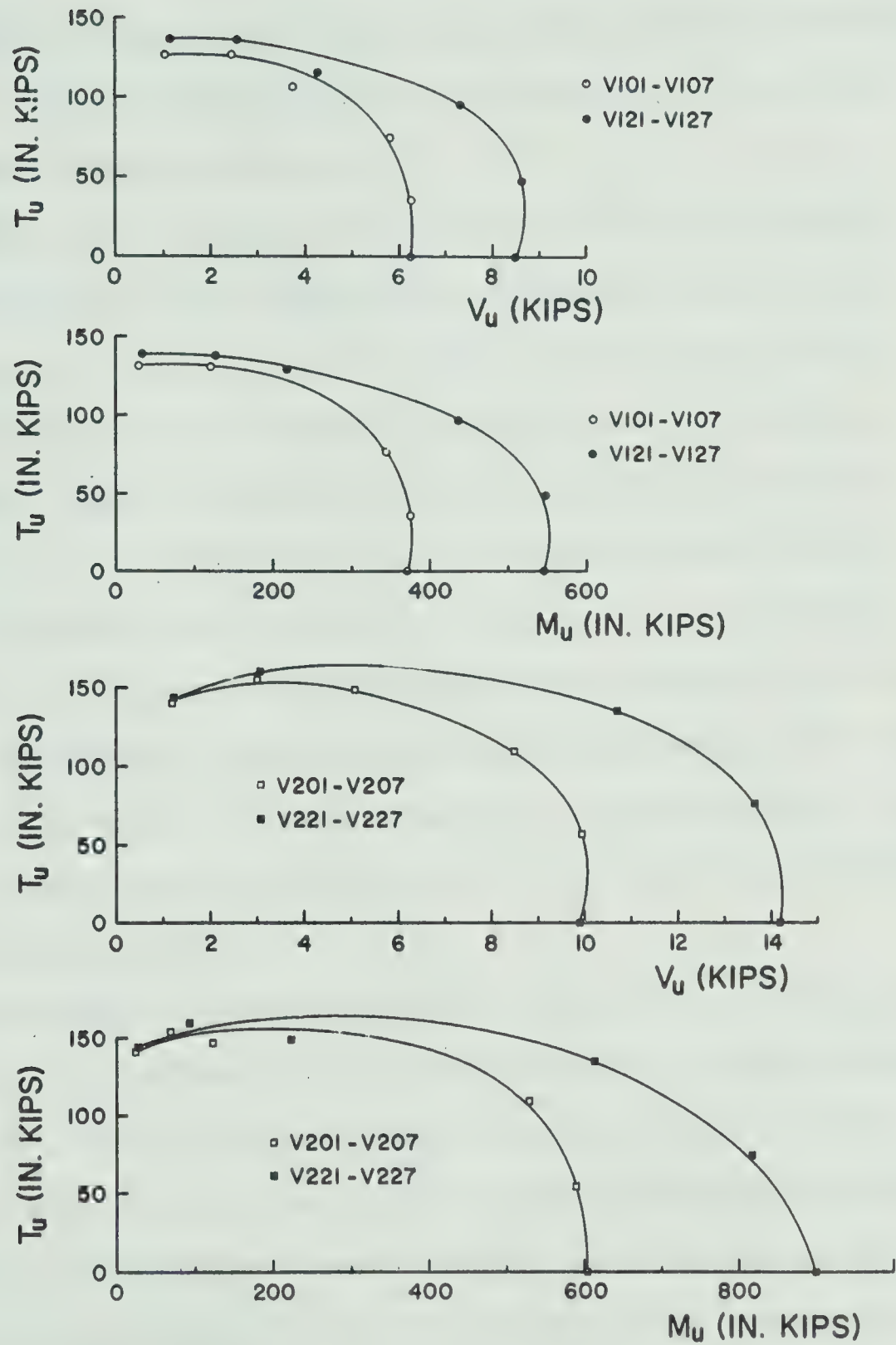


FIGURE 5.4 INTERACTION DIAGRAMS



capacity of the section beyond the cracking stage up to the ultimate loading condition. Dimensional interaction diagrams shown in FIGURE 5.4 are used to illustrate the effect on the beam behavior of varying the torsional moment to bending moment ratio.

In the case of bending and shear with little or no torsion, the bottom longitudinal steel experienced high tensile strains in resisting the applied flexural moment. Cracks occurred on the bottom of the specimen and continued vertically upwards defining the lower boundary of the flexural compression zone. Under increased loading, these cracks angled in towards the load point stopping approximately one-third of the depth down from the top. The failure plane occurred quite close to the load point and crushing of the concrete along the top fibre was quite evident. Examination of the test specimens after failure revealed that the length over which cracking occurred was greater in the case of the higher prestress force because of the higher bending moment needed to bring about failure.

In the medium range of torque to bending ratio, cracking occurred on the bottom and side faces almost simultaneously. The crack pattern became quite inclined and failure occurred along an inclined crack at some distance from the load point. The transverse reinforcement provided a large amount of the torsional stiffness after cracking and as failure approached, the top longitudinal steel developed tensile strains due to the twisting action of the torsional couple. It can be seen from the torque-twist curves in APPENDIX A that the available rotation capacity



is reduced with an increase in the degree of prestress, and also with an increase in the proportion of bending moment. It appeared, that for the same degree of prestress force, the beams which were eccentrically prestressed exhibited greater ductility than those which were prestressed concentrically. This is illustrated in the shape of the moment-deflection diagrams in APPENDIX A. Because the internal resisting couple possessed a large lever arm due to the location of the strands below the centroid of the cross section, these specimens had a greater flexural capacity than those prestressed concentrically. Therefore, provided that the prestress force is low enough to ensure a tensile-type of failure, the eccentrically prestressed beams will exhibit greater ductility.

As the proportion of torsional moment was increased, the cracks developed near mid-height and at a considerable distance from the load point. The four gauges located on the stirrups in the gauge length indicated large strains were taking place. Inclined cracking progressed along the length of the beam as the loading was increased. A particularly wide crack with some evidence of crushing in the concrete indicated the plane on which failure occurred. In the case of the higher prestress level, the inclination of the torsional cracking was decreased and the pattern became more horizontal in nature due to the higher initial compressive stress in the concrete.

Individual beam behavior can be analyzed using the test results presented in the beam tables in APPENDIX A. Two methods of graphic representation have been used to present these results; these are the





torque-twist curves and the moment-deflection curves shown in APPENDIX A. In plotting the moment-deflection diagrams, the coordinates used were the maximum flexural moment under the load and the deflection of the west gauge. Because in every case the failure plane was situated at a distance removed from the load point, the moment at the failure section at ultimate,  $M_u$ , always existed in an area subjected to transverse shear stresses, whereas theoretically at least, no shear stresses were present at the load point section. Hence, the value of the moment existing at the load point section at ultimate was always greater than the moment existing at the failure plane section. Strictly speaking, the transverse load was distributed to the specimen over a 6 inch length, and hence, there existed a steep shear gradient over this small portion of the beam.

For the specimens subjected to a greater proportion of flexural moment with respect to torsional moment, it would appear from analyzing the bottom longitudinal steel strains and the general shape of the moment-deflection curves, that the yield strain in the mild steel reinforcement was reached and that a tensile-type of failure occurred with adequate ductility exhibited prior to failure. In the case of the eccentrically prestressed beams, the strands possibly reached their yield strain and contributed to the post-cracking ductility of the specimen. However, those prestressed concentrically exhibited less ductility and it would appear that, because of their location in the section, the strands did not attain the same strain as those positioned eccentrically. When the proportion of torsional moment with respect to bending moment was increased,



the transverse and longitudinal reinforcement mainly provided the rotational resistance after cracking. Prestressing eccentrically added to the available rotational capacity, but prestressing concentrically afforded little increase in this respect.

### 5-3 GENERAL EFFECT OF PRESTRESSING

Because the prestress force introduces initial compressive stresses in the concrete, its effect is to increase the resistance of the section to applied torsion, shear and bending. The effective tensile strength of the concrete is increased with prestressing with the result that cracking is delayed until such time in the loading sequence that this initial compressive stress has been neutralized. It has been found that, for high prestress levels, the amount of post-cracking behavior is reduced; hence, the beams have the tendency to fail in a rather brittle manner with a small amount of ductility exhibited after initial cracking.

#### (a) Effect of Degree of Prestress

The effect of using a higher level of prestress can best be visualized by referring to the top diagrams in FIGURE 5.1 and FIGURE 5.2. In the case of combined bending and shear, the capacity at ultimate for Beam V201, while still less than that for pure bending, is greater than the value found for Beam V101, and this increase must be due to the higher value of prestress present. In the case of high torque to moment ratio, i.e. Beam V205, the torsional capacity under combined loading is still less than under pure torsional loading, but greater than that value for the case of the lower prestressed Beam V105. Hence, the detrimental



effect of the torsional and transverse shear stresses is slightly lessened with the introduction of higher initial compressive stress. But as the torque to bending ratio is reduced, the value of the torsional moment at failure is greater than the capacity of the section in pure torsion. This increase can be greatly attributed to the higher level of prestress as the presence of flexure does not significantly increase the resistance to torsional stresses particularly for concentrically prestressed beams as exemplified in the curve for Beams V101-V107 in FIGURE 5.1. This beneficial effect can also be seen by comparing the top and bottom diagrams in FIGURE 5.3.

#### (b) Effect of Location of Prestress

The effect of prestressing eccentrically can be seen by comparing the top and bottom diagrams in FIGURE 5.1 and FIGURE 5.2. For a beam subjected to bending and shear only, the capacity of the eccentrically prestressed section exceeds that for the specimen prestressed concentrically regardless of the degree of prestress. One of the more important findings as a result of the tests performed by Sorensen (12) was that a small amount of bending was beneficial to the torsional strength as shown in the bottom diagram of FIGURE 5.1. However, for our case, a small amount of bending had a negligible effect on the torsional strength. It would appear that the introduction of transverse shear stresses into the section counterbalanced any helpful effect that a small amount of bending might afford. Examination of the moment-deflection curves in APPENDIX A shows that prestressing eccentrically has the desirable effect





of producing a more ductile type of failure even for the case of higher prestress.

#### 5-4 EFFECT OF TRANSVERSE SHEAR

FIGURE 5.1 and FIGURE 5.2 illustrates the effect of transverse shear by comparing the results of two series of tests; one including shear and the other without. It can be seen that, for the case of combined bending and shear, the presence of shear reduces the ultimate capacity in flexure that the specimen is capable of sustaining regardless of the degree and location of the prestress. For the case of combined bending, torsion and shear, the effect of shear is to reduce the ultimate torsional capacity of a section under a lower degree of prestress regardless of its location as shown in the top diagram of FIGURE 5.3.



## CHAPTER VI

### SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

#### 6-1 INTRODUCTION

This chapter includes a summary of the test results, general conclusions resulting from the investigation, and recommendations for further studies of this nature.

#### 6-2 SUMMARY

For this study, twenty four rectangular concrete beams were tested. Identical longitudinal and transverse reinforcement were provided for all specimens. Twelve were prestressed concentrically while the others were prestressed eccentrically at an eccentricity ratio of 0.167. Also, two degrees of prestress force were studied, thus dividing the beams into four series. At the lower degree of prestress, Beams V101-V107 were concentrically prestressed while Beams V121-127 were prestressed eccentrically. Beams V201-V207 were concentrically prestressed while Beams V221-V227 were prestressed eccentrically at the higher degree of prestress.

All beams exhibited two stages in their behavior and these are designated as pre-cracking and post-cracking. Essentially elastic behavior existed prior to cracking which was terminated by major cracking at that point indicated by the abrupt change in slope in either the moment-deflection curves or the torque-twist curves. The mild steel reinforce-



ment provided the post-cracking ductility exhibited by all the specimens.

The test results have been presented in the form of tabulated values, TORQUE-TWIST curves, MOMENT-DEFLECTION curves, INTERACTION DIAGRAMS, and discussions. The interaction diagrams have been shown in both dimensional and non-dimensional form for the ultimate loading condition. These diagrams graphically present a summary of the test results illustrating the effects of both types of prestressing, two degrees of prestressing, the torsional moment to flexural moment ratio, and the effect of transverse shear at the ultimate loading stage.

### 6-3 CONCLUSIONS

In order to study the different variables, the twenty four specimens were divided into four groups of six beams. Hence, the results of this investigation are based upon a limited number of tests, and should be interpreted as such.

From the test results, it is concluded that:

(1) Concentric prestressing increases the cracking capacity of a rectangular concrete section under combined bending, shear and torsion by increasing the effective tensile strength of the concrete.

(2) The ultimate capacity of an eccentrically prestressed rectangular concrete section subjected to shear and bending is greater than that for the same specimen prestressed concentrically. However, for the case of combined bending, shear and torsion, the type of prestress seems to have little effect on ultimate capacity.

(3) The ultimate capacity of a rectangular concrete section sub-





jected to bending, shear and torsion is increased by using a higher level of prestress force. Generally, the effect of prestressing is beneficial provided that the level of prestress force is not great enough to alter the failure mode from one of principal tension to a compression-type failure.

(4) The effect of transverse shear is to reduce the ultimate capacity of a prestressed rectangular concrete section subjected to either bending and shear, or bending and shear combined with torsion. Because the transverse shear introduces tensile stresses into the section, its effect is to bring about failure at a smaller load.

(5) The detrimental effect of the transverse shear stresses can be overcome by prestressing with a higher degree of prestress force. This can be achieved regardless of the type of prestressing used. The ultimate torsional capacity of the section can even be increased beyond that for the case of pure torsion but this higher capacity is still slightly less than that for the case of combined bending and torsion; hence, the effect of the shearing stresses is not completely nullified.

#### 6-4 RECOMMENDATIONS

The following recommendations are made for the benefit of future investigations:

(1) Comparisons between prestressed sections and identical non-prestressed specimens should be established in order to verify the effect of prestress on combined load capacity.

(2) More test data is needed for torque to bending moment ratios



of the order of 0 to 0.33 as this is probably the approximate ratio of loading found in practice.

(3) Varying amounts of mild steel reinforcement should be included in future tests to determine how its amount and distribution across the section affects the beam behavior.

(4) Sections other than rectangular should be tested as prestressed concrete is generally employed as 'I' and 'T' sections in practice.



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APPENDIX A  
TEST RESULTS



LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS IN. $\times 10^3$			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
					EAST	CENTER	WEST	1	2	3	4	5	6
0	-	6.5	0.09	-	0	0	0	0	0	0	0	0	0
1	-	48	0.67	-	0	20	25	3	2	7	7	45	-27
2	-	96	1.33	-	10	40	45	5	3	3	0	86	-56
3	-	144	2.00	-	20	60	75	2	1	1	0	136	-100
4	-	192	2.67	-	30	85	105	5	1	-6	-5	203	-146
5	-	240	3.33	-	40	110	145	4	3	-2	+43	281	-196
6	-	264	3.67	-	50	130	170	5	3	-1	81	335	-221
7	-	288	4.00	-	60	155	200	3	5	-7	94	394	-254
8	-	312	4.33	-	70	185	235	5	5	-9	112	577	-286
9	-	336	4.67	-	80	215	265	3	5	+1	122	740	-320
10	-	360	5.00	-	90	245	315	5	7	45	125	951	-354
11	-	384	5.33	-	110	290	375	7	9	68	121	1149	-395
12	-	408	5.67	-	130	340	440	4	14	93	121	1329	-440
13	-	432	6.00	-	160	410	540	3	14	119	148	1543	-499
14	-	449	6.23	-									

TABLE A.1 BEAM 101

NOTE: DEFLECTIONS POSITIVE DOWNWARD

POSITIVE STRAIN INDICATES TENSION





LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN.KIP	SHEAR KIP	TWIST RAD/IN.  x 10 <sup>6</sup>	DEFLECTIONS			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE							
					EAST	CENTER IN. x 10 <sup>3</sup>	WEST	1	2	3	4	5			
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0	0	0
1	13.0	72	1.00	13	10	25	30	7	16	8	27	104			
2	17.3	96	1.33	19	15	30	35	7	15	7	28	127			
3	21.7	120	1.67	25	15	40	45	3	14	5	23	135			
4	26.0	144	2.00	29	20	50	60	0	19	10	35	160			
5	30.3	168	2.33	36	25	65	75	-2	14	5	55	200			
6	34.7	192	2.67	46	30	80	100	-2	17	-3	78	231			
7	39.0	216	3.00	57	35	95	120	+3	19	0	105	274			
8	43.3	240	3.33	71	40	115	150	4	15	-31	225	331			
9	47.7	264	3.67	96	55	145	190	14	13	-47	451	437			
10	49.8	276	3.83	113	60	165	210	28	12	-64	586	473			
11	52.0	288	4.00	126	65	180	230	38	12	-68	713	576			
12	54.2	300	4.17	139	65	190	250	56	18	-68	824	650			
13	56.3	312	4.33	160	75	205	265	81	18	-57	955	825			
14	58.5	324	4.50	177	80	220	290	131	27	-31	1055	953			
15	60.7	336	4.66	198	85	240	310	163	50	+9	1163	1089			
16	62.8	348	4.83	217	95	260	335	360	86	47	1240	1208			
17	65.0	360	5.00	256	100	285	370	733	142	92	1359	1357			
18	67.2	372	5.17	289	115	320	415	822	234	131	1525	1477			
19	69.4	384	5.33	327	125	350	460	910	347	160	1767	1587			
20	71.5	396	5.50	392	145	405	540	1003	512	212	2007	1700			
21	73.7	408	5.67	470	170	470	630	1074	600	264	2517	1790			
22	75.9	418	5.79	699				798	502	370	5290	1460			

TABLE A.2 BEAM 102



LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN.KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS			REINFORCEMENT STRAINS					
					EAST	CENTER IN. $\times 10^3$	WEST	1	2	3	4	5	6
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0
1	14.6	36	0.50	47	0	5	10	2	-7	-13	0	9	-26
2	19.5	48	0.67	54	5	10	15	7	-7	-14	3	24	-36
3	24.4	60	0.83	60	5	10	20	7	-6	-11	7	40	-45
4	29.2	72	1.00	68	5	15	20	11	-5	-13	9	49	-55
5	34.1	84	1.17	74	10	20	25	14	-4	-14	14	60	-66
6	39.0	96	1.33	80	10	25	35	20	-3	-12	15	75	-73
7	43.9	108	1.50	86	10	25	40	26	+5	-8	24	90	-86
8	48.7	120	1.67	99	10	30	40	38	6	0	34	103	-94
9	53.6	132	1.83	107	15	35	45	65	12	8	44	117	-107
10	58.5	144	2.00	121	15	40	55	92	30	17	60	138	-114
11	63.4	156	2.17	138	20	50	70	164	74	32	66	156	-128
12	68.2	168	2.33	164	25	60	85	219	119	23	62	166	-147
13	73.1	180	2.50	203	25	75	95	248	188	25	57	186	-158
14	78.0	192	2.67	268	30	95	120	315	319	401	73	252	-157
15	79.9	197	2.73	308	35	100	135	364	407	539	89	316	-141
16	81.9	202	2.80	346	35	105	140	425	526	671	128	414	-123
17	83.8	206	2.87	399	40	115	150	501	723	793	184	550	-78
18	85.8	211	2.93	449	40	125	160	651	875	897	247	655	-52
19	87.7	216	3.00	500	45	135	170	781	966	969	293	758	-44
20	89.7	221	3.07	564	50	150	185	910	1055	1028	341	886	-30
21	91.6	226	3.13	628	50	160	195	1004	1133	1088	421	1022	-1
22	93.6	230	3.20	704	50	165	205	1072	1170	1152	513	1167	+26
23	95.5	235	3.27	755	55	180	215	1127	1211	1223	597	1275	45
24	97.5	240	3.33	824	60	185	230	1205	1252	1335	698	1392	70
25	99.5	245	3.40	914	55	185	240	1285	1346	1620	811	1548	94
26	101.4	250	3.47	992	55	205	250	1354	1401	2171	914	1692	116
27	103.4	254	3.53	1074	60	215	270	1358	1498	2340	986	1864	143
28	105.3	259	3.60	1173	70	230	285	1229	1599	2631	1060	2058	168
29	107.3	264	3.67	1279	75	250	305	1273	1718	3420	1140	2214	193
30		269	3.73	1672	75	305	380	1312	1897	13538	1644	2406	256

TABLE A.3 BEAM 103



LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. x 10 <sup>6</sup>	DEFLECTIONS			REINFORCEMENT STRAINS								
					EAST	CENTER IN. x 10 <sup>3</sup>	WEST	MICRO INCHES PER INCH GAUGE								
								1	2	3	4	5	6			
0	0	6.5	0.09	0	0	0	0	0	0	0						0
1	17.3	24	0.33	13	5	10	20	1	2	5						12
2	34.7	48	0.67	34	10	20	30	7	10	12						32
3	52.0	72	1.00	60	10	25	35	35	29	35						59
4	60.7	84	1.17	75	10	30	45	60	43	58						75
5	69.3	96	1.33	96	10	40	45	90	56	99						94
6	78.0	108	1.50	124	15	40	55	138	130	185						118
7	86.7	120	1.67	155	15	45	60	192	264	315						146
8	90.1	125	1.73	180	15	55	70	221	320	480						166
9	93.6	130	1.80	211	20	60	75	261	400	584						185
10	97.1	134	1.87	265	20	65	85	391	586	751						212
11	100.5	139	1.93	393	20	80	100	560	875	967						331
12	104.0	144	2.00	511	15	85	115	886	1062	1102						501
13	105.7	146	2.03	593	20	85	115	1162	1184	1198						619
14	107.5	149	2.07	654	20	90	120	1335	1265	1275						704
15	109.2	151	2.10	711	20	100	125	1483	1327	1333						765
16	110.9	154	2.13	769	15	100	135	1596	1394	1373						823
17	112.7	156	2.17	833	20	110	140	1580	1432	1408						880
18	114.4	158	2.20	897	20	115	145	1556	1494	1445						938
19	116.1	161	2.23	969	20	115	150	1900	1540	1468						1008
20	117.9	163	2.27	1036	20	125	155	3312	1577	1443						1085
21	119.6	166	2.30	1104	20	125	165	-	1394	183						-34
22	121.3	168	2.33	1178	25	125	170	-	1374	144						+18
23	123.1	170	2.37	1270	20	130	175	-	1328	276						103
24	124.8	173	2.40	1391	25	135	185	-	1300	272						225
25	126.6	175	2.43	1570	25	145	195	-	1273	387						384
26	128.3	178	2.47	1896	35	165	225	-	4050	9625						622
27	128.3	178	2.47	2152	35	175	245	-	8890	-						-700
28	128.3	178	2.47	2573	545	185	295	-	9950	-						-821

TABLE A.4 BEAM 104





LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
					EAST	CENTER $\text{IN.} \times 10^3$	WEST	1	2	3	4	5	6
0	0	6.5	0.09	0	0	0	0	0	0	0	0		0
1	19.5	12	0.17	21	0	0	5	5	6	0	225		-1
2	39.0	24	0.33	45	0	0	5	12	7	3	231		-14
3	46.8	29	0.40	56	0	5	10	18	12	13	239		-17
4	54.6	34	0.47	69	0	5	10	32	19	27	249		-22
5	62.4	38	0.53	83	0	10	10	61	34	55	258		-26
6	70.2	43	0.60	101	0	5	15	153	46	91	294		-34
7	78.0	48	0.67	120	0	5	15	257	67	167	375		-34
8	85.8	53	0.73	155	-5	5	15	353	125	252	488		-24
9	93.6	58	0.80	241	0	15	25	893	242	372	623		+59
10	97.5	60	0.83	367	-5	20	30	1049	827	812	650		344
11	101.4	62	0.87	503	-5	20	40	1256	981	1204	1400		390
12	105.3	65	0.90	597	-10	25	40	1326	1090	1376	3250		434
13	109.2	67	0.93	700	-10	30	45	1342	1179	1472	3664		468
14	113.1	70	0.97	835	-10	30	50	1350	1303	1614	10660		502
15	117.0	72	1.00	966	-10	35	60	1276	1373	1750	13239		580
16	120.9	74	1.03	1117	-10	35	65	1200	1448	1861	14939		709
17	124.8	77	1.07	1352	-5	35	65	1190	1554	2000	16870		890
18	124.8	79	1.10	1701	-5	35	75	1170	1590	4465	14785		1091
19	128.7	79	1.10	2115	-10	25	80	1185	1779	17218	13992		1235
20	128.7	79	1.10	2497	-5	45	95	1022	1828	19491	13110		1176

TABLE A.5 BEAM 105



TABLE A.6 BEAM 107

LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS			REINFORCEMENT STRAINS MICRO INCHES PER INCH					
					EAST	IN. $\times 10^3$ CENTER	WEST	GAUGE					
								1	2	3	4	5	6
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0
1	9.9	120	1.67	6	15	35	40	-9	-5	-5	-1	102	-105
2	11.1	144	2.00	7	20	50	50	-6	-7	-7	-3	126	-125
3	13.0	168	2.33	10	20	60	65	-7	-8	-8	-4	154	-152
4	13.9	180	2.50	14	25	65	75	-10	-6	-3	-4	167	-160
5	14.9	192	2.67	14	30	75	85	-8	-6	-8	-6	197	-175
6	15.8	204	2.83	14	30	80	95	-2	-2	-4	-7	234	-188
7	16.7	216	3.00	18	30	85	105	+3	-3	-7	-4	252	-206
8	17.6	228	3.17	19	35	95	115	-2	-4	-9	+3	283	-214
9	18.6	240	3.33	22	40	110	130	+2	-4	-10	-5	303	-235
10	19.5	252	3.50	25	45	120	145	3	-5	-14	-4	335	-247
11	20.4	264	3.67	28	45	125	155	1	-5	-16	+8	371	-266
12	21.3	276	3.83	32	50	140	170	8	-4	-13	17	400	-281
13	22.3	288	4.00	35	55	150	185	4	-5	-10	24	434	-298
14	23.2	300	4.17	39	60	160	205	5	-6	-6	26	453	-321
15	24.1	312	4.33	42	65	175	220	0	-10	-12	26	498	-340
16	25.1	324	4.50	46	65	190	240	4	-12	-8	28	567	-363
17	26.0	336	4.67	50	75	205	260	0	-18	-10	37	639	-385
18	26.9	348	4.83	57	80	220	275	5	-21	-3	50	756	-411
19	27.9	360	5.00	60	85	235	290	3	-24	+6	57	846	-433
20	28.8	372	5.17	67	95	255	315	6	-22	15	80	960	-459
21	29.7	384	5.33	74	100	275	345	7	-24	25	88	1048	-483
22	30.7	396	5.50	78	110	300	375	17	-24	49	111	1138	-507
23	31.6	408	5.67	88	115	325	415	13	-29	64	123	1223	-529
24	32.5	420	5.83	97	125	355	445	13	-34	86	138	1315	-556
25	33.4	432	6.00	110	140	390	500	11	-37	102	165	1414	-578
26	34.4	444	6.17	125	165	480	620	18	-35	121	205	1520	-604
27	35.3	456	6.33		280	775	945	18	-25	136	942	-	-



LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS		REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
					EAST	WEST	1	2	3	4	5	6
0	-	6.5	0.09	-	0	0	0	0	0	0	0	
1	-	48	0.67	-	5	10	-2	0	-1	-1	29	
2	-	96	1.33	-	10	30	-1	0	0	-2	64	
3	-	144	2.00	-	15	50	0	0	0	-2	100	
4	-	192	2.67	-	20	70	0	0	0	-2	152	
5	-	240	3.33	-	30	90	0	0	0	-4	201	
6	-	288	4.00	-	40	125	0	0	0	-3	265	
7	-	336	4.67	-	60	170	0	0	-5	-3	324	
8	-	360	5.00	-	60	190	0	0	-6	-4	359	
9	-	384	5.33	-	70	220	0	0	-8	0	392	
10	-	408	5.67	-	80	250	0	0	-11	10	429	
11	-	432	6.00	-	85	275	0	0	-11	13	470	
12	-	456	6.33	-	95	305	0	0	-17	10	540	
13	-	480	6.67	-	100	330	0	-4	-24	10	718	
14	-	504	7.00	-	105	365	0	-6	-30	10	880	
15	-	528	7.33	-	110	405	0	-10	-34	19	1068	
16	-	552	7.67	-	120	460	0	-11	-36	23	1242	
17	-	564	7.83	-	130	500	0	-10	-34	23	1312	
18	-	576	8.00	-	140	540	0	-10	-34	25	1385	
19	-	588	8.17	-	150	575	0	-8	-32	31	1460	
20	-	600	8.33	-	160	635	0	-6	-30	58	1543	
21	-	612	8.50	-	280	990	0	-5	-20	1200	1635	

TABLE A.7 BEAM 121





LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN.KIP	SHEAR KIP	TWIST RAD/IN. x 10 <sup>6</sup>	DEFLECTIONS			REINFORCEMENT STRAINS					
					EAST	CENTER IN. x 10 <sup>3</sup>	WEST	MICRO INCHES PER INCH GAUGE					
								1	2	3	4	5	6
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0
1	13.0	72	1.00	6	10	15	20	8	6	6	8	53	-58
2	17.3	96	1.33	11	20	25	30	12	6	6	6	69	-85
3	21.7	120	1.67	15	20	30	45	10	8	8	8	87	-116
4	26.0	144	2.00	19	25	40	55	10	8	8	8	104	-143
5	30.3	168	2.33	26	30	45	65	10	10	10	8	126	-178
6	34.7	192	2.67	33	35	55	70	10	14	14	8	146	-206
7	39.0	216	3.00	36	40	70	85	8	12	12	8	170	-242
8	43.3	240	3.33	49	40	75	100	8	14	14	12	191	-275
9	47.7	264	3.67	60	50	90	120	8	18	18	16	220	-316
10	52.0	288	4.00	68	55	105	140	14	29	29	24	248	-350
11	56.3	312	4.33	87	65	125	165	12	33	33	124	281	-402
12	60.7	336	4.67	107	65	145	190	15	45	45	324	312	-444
13	65.0	360	5.00	139	75	180	225	14	64	64	607	346	-500
14	67.2	372	5.17	151	80	185	240	14	116	116	720	372	-521
15	69.4	384	5.33	171	85	200	260	14	260	260	810	598	-555
16	71.5	396	5.50	197	95	220	275	-5	423	423	926	695	-570
17	73.7	408	5.67	225	100	230	300	-8	535	535	1015	770	-608
18	75.9	420	5.83	257	105	255	315	+4	654	654	1118	857	-629
19	78.0	432	6.00	294	105	270	340	260	778	778	1214	1047	-630
20	80.2	444	6.17	342	115	285	360	468	893	893	1332	1164	-600
21	82.4	456	6.33	379	125	305	385	550	982	982	1430	1260	-558
22	84.5	468	6.50	431	130	325	405	646	1057	1057	1581	1372	-515
23	86.7	480	6.67	482	140	345	440	693	1144	1144	1712	1486	-506
24	88.9	492	6.83	557	155	375	470	740	1224	1224	1855	1612	-490
25	91.0	504	7.00	624	160	415	525	784	1324	1324	1968	1736	-505
26	93.2	516	7.17	667	170	455	575	821	1380	1380	2027	1836	-515
27	95.4	528	7.33	846	240	655	850	852	1386	1386	1923	1794	-588
28	86.4	528	7.33	1133	335	955	1225	847	1412	1412	2022	1762	-610

TABLE A.8 BEAM 122



LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN.KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS IN. $\times 10^3$		REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
					EAST	CENTER WEST	1	2	3	4	5	6
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0
1	14.6	36	0.50	8	5	5	-8	-9	-10	18	18	-22
2	19.5	48	0.67	17	5	10	-10	-10	-11	29	29	-31
3	24.4	60	0.83	19	10	10	-10	-10	-14	33	33	-46
4	29.2	72	1.00	26	10	20	-12	-10	-17	48	48	-60
5	34.1	84	1.17	33	10	20	-14	-9	-19	58	58	-69
6	39.0	96	1.33	40	10	30	-14	-6	-20	65	65	-80
7	43.9	108	1.50	49	10	35	-13	-2	-20	78	78	-91
8	48.7	120	1.67	54	15	40	-12	-1	-20	88	88	-109
9	53.6	132	1.83	64	10	40	-9	0	-20	100	100	-120
10	58.5	144	2.00	76	15	45	0	8	-22	111	111	-140
11	63.4	156	2.17	86	20	50	7	19	-20	129	129	-153
12	68.2	168	2.33	99	25	55	16	30	-17	145	145	-166
13	73.1	180	2.50	108	25	60	30	54	-15	161	161	-183
14	78.0	192	2.67	129	25	70	49	102	-8	185	185	-196
15	82.9	204	2.83	150	30	80	70	189	-14	219	219	-211
16	87.7	216	3.00	188	30	90	88	243	-31	269	269	-222
17	92.6	228	3.17	279	40	110	101	299	-9	331	331	-224
18	97.5	240	3.33	511	45	130	76	925	+284	749	749	-207
19	99.4	245	3.40	611	45	140	170	1064	340	913	913	-191
20	101.4	250	3.47	665	45	145	460	1129	365	980	980	-200
21	103.3	254	3.53	696	50	155	845	1159	381	1060	1060	-220
22	105.3	259	3.60	839	55	160	1391	1226	420	1131	1131	-237
23	107.2	264	3.67	908	50	170	1510	1269	464	1200	1200	-241
24	109.2	269	3.73	979	60	175	1568	1320	528	1275	1275	-231
25	111.1	274	3.80	1075	55	180	1557	1356	590	1361	1361	-222
26	113.1	278	3.87	1158	60	190	1496	1390	647	1440	1440	-215
27	115.0	283	3.93	1244	65	200	1455	1430	681	1529	1529	-204
28	117.0	288	4.00	1349	60	205	1462	1450	730	1647	1647	-206
29	119.0	293	4.07	1460	65	210	1480	1512	780	1771	1771	-171
30	120.9	298	4.13	1593	65	215	1465	1521	841	1908	1908	-124
31	122.9	302	4.20	1721	70	230	1505	1548	917	2000	2000	-79
32	124.8	307	4.27	1907	80	240	1578	1593	1208	2080	2080	-1
33	126.8	312	4.33	2389	80	265	1412	1531	1526	2083	2083	+100

TABLE A.9 BEAM 123





LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS IN. $\times 10^3$			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
					EAST	CENTER	WEST	1	2	3	4	5	6
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0
1	17.3	24	0.33	15	0	5	5	-2	-1	2	0	0	-15
2	34.7	48	0.67	36	0	10	10	-8	-2	5	1	0	-42
3	43.3	60	0.83	47	0	15	15	-9	0	4	4	0	-55
4	52.0	72	1.00	58	5	15	20	-10	2	5	7	0	-66
5	60.7	84	1.17	74	5	20	25	-9	0	8	11	0	-79
6	69.3	96	1.33	90	5	20	30	-3	8	19	15	0	-95
7	78.0	108	1.50	110	5	25	35	+10	32	72	20	0	-105
8	86.7	120	1.67	138	5	30	40	55	61	214	27	0	-109
9	95.3	132	1.83	176	5	35	50	131	110	420	87	0	-110
10	104.0	144	2.00	390	5	45	60	348	376	1137	490	0	-56
11	107.4	149	2.07	533	5	50	70	700	526	1250	633	0	-26
12	110.9	154	2.13	715	0	50	75	848	1225	1313	775	0	+206
13	114.3	158	2.20	857	-5	50	80	931	1397	1590	946	0	364
14	117.8	163	2.26	969	-5	50	90	997	1497	1623	1113	0	482
15	121.3	168	2.33	1086	-5	50	90	1052	1565	1546	1233	0	631
16	123.0	170	2.37	1197	-5	50	95	1116	1596	1472	1319	0	870
17	124.8	173	2.40	1296	-5	45	100	1156	1632	1458	1393	0	1066
18	126.5	175	2.43	1382	-10	50	105	1206	1622	1488	1466	0	1201
19	128.2	178	2.47	1474	-10	45	115	1260	1638	1507	1525	0	1318
20	130.0	180	2.50	1585	-15	45	115	1335	1576	1500	1594	0	1460
21	131.7	182	2.53	1679	-10	50	120	1374	1588	1484	1668	0	1574
22	133.4	185	2.57	1819	-15	40	120	1420	1576	1535	1781	0	1759
23	135.2	187	2.60	1960	-15	40	125	1467	1581	1547	1859	0	1935
24	136.9	190	2.63	2138	-10	45	140	1440	1589	1548	1911	0	2101
25	136.9	192	2.67	2528	-10	45	160	1420	1549	1458	2055	0	1995
26	136.9	192	2.67	2783	-10	25	170	1299	1396	1316	1973	0	1790

TABLE A.10 BEAM 124





LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
					EAST	CENTER $\text{IN.} \times 10^3$	WEST	1	2	3	4	5	6
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0
1	19.5	12	0.17	18	0	0	5	2	-8	-5	-8	23	-17
2	39.0	24	0.33	40	5	0	5	7	-10	-7	-10	35	-23
3	46.8	29	0.40	50	0	5	5	13	-10	-6	-11	45	-35
4	54.6	34	0.47	60	0	0	5	20	-10	0	-14	48	-39
5	62.4	38	0.53	71	0	0	5	27	-8	-7	-15	58	-40
6	70.2	43	0.60	85	0	0	10	40	-2	-6	-12	65	-49
7	78.0	48	0.67	100	0	5	10	61	+3	0	-10	75	-55
8	81.9	50	0.70	110	-5	0	5	76	8	6	-10	81	-52
9	85.8	53	0.73	119	-5	5	5	97	9	17	-6	88	-53
10	89.7	55	0.77	129	-10	0	10	131	13	23	-8	93	-57
11	93.6	58	0.80	142	-10	5	5	184	19	24	-6	103	-45
12	97.5	60	0.83	156	-10	0	5	212	28	32	-3	110	-32
13	101.4	62	0.87	175	-10	0	10	250	119	40	-2	120	+15
14	105.3	65	0.90	204	-15	0	5	291	207	45	+5	127	37
15	109.2	67	0.93	342	-15	-10	0	570	870	36	0	155	412
16	113.1	70	0.97	451	-20	-20	0	1127	1030	431	83	372	491
17	117.0	72	1.00	626	-25	-25	-5	1295	1119	847	146	532	519
18	120.9	74	1.03	771	-20	-20	0	1434	1215	1060	316	689	568
19	124.8	77	1.07	893	-25	-25	5	1520	1268	1197	447	828	625
20	128.7	79	1.10	1039	-35	-35	0	1600	1301	1248	647	960	647
21	132.6	82	1.13	1233	-35	-40	0	1610	1590	1196	819	1165	570
22	136.5	84	1.17	1514	-40	-55	5	1648	4458	1147	944	1396	354
23	136.5	84	1.17	1774	-35	-65	10	1574	7080	1175	948	1562	-204
24	136.5	86	1.20	2296	-35	-105	10	1560	4710	14205	897	1728	-833
25	136.5	86	1.20	2700	-35	-160	15	1530	6270	18691	857	1683	-1159

TABLE A.11 BEAM 125



LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS			REINFORCEMENT STRAINS							
					EAST	IN. $\times 10^3$ CENTER	WEST	MICRO INCHES PER INCH GAUGE							
								1	2	3	4	5	6		
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0	0	0
1	10.1	120	1.67	8	10	35	35	-10	-10	-7	-7	91	-100	91	-100
2	11.1	144	2.00	8	10	40	45	-10	-10	-9	-11	114	-128	114	-128
3	13.0	168	2.33	11	15	50	55	-10	-10	-11	-13	140	-151	140	-151
4	14.9	192	2.67	14	20	60	65	-10	-10	-8	-13	163	-177	163	-177
5	16.7	216	3.00	18	20	70	85	-10	-10	-10	-20	192	-202	192	-202
6	18.6	240	3.33	21	25	80	95	-11	-12	-11	-21	226	-231	226	-231
7	20.4	264	3.67	24	35	90	110	-11	-10	-14	-23	254	-263	254	-263
8	22.3	288	4.00	28	40	105	125	-13	-9	-13	-30	289	-292	289	-292
9	24.1	312	4.33	32	40	120	150	-15	-10	-18	-49	318	-327	318	-327
10	26.0	336	4.67	38	50	135	170	-17	-10	-20	-49	361	-365	361	-365
11	27.9	360	5.00	42	55	155	195	-18	-14	-22	-42	409	-402	409	-402
12	29.7	384	5.33	50	60	180	220	-19	-15	-14	-23	450	-447	450	-447
13	31.6	408	5.67	58	70	200	250	-20	-16	+13	0	538	-488	538	-488
14	33.4	432	6.00	67	80	225	280	-21	-17	39	21	719	-541	719	-541
15	35.3	456	6.33	79	90	250	310	-22	-21	60	50	1028	-595	1028	-595
16	37.2	480	6.67	90	100	275	340	-23	-17	75	80	1241	-645	1241	-645
17	38.1	492	6.83	99	105	295	365	-24	-10	87	91	1341	-670	1341	-670
18	39.0	504	7.00	106	110	310	385	-27	-10	100	102	1444	-692	1444	-692
19	39.9	516	7.17	113	115	325	405	-26	-11	113	119	1539	-720	1539	-720
20	40.9	528	7.33	122	125	340	430	-29	-11	127	138	1639	-746	1639	-746
21	41.8	540	7.50	133	130	370	460	-30	-9	149	181	1739	-769	1739	-769
22	42.7	552	7.67	146	135	385	490	-30	-6	168	221	1831	-794	1831	-794
23	43.7	564	7.83	156	145	410	525	-31	-5	193	244	1915	-821	1915	-821
24	44.6	576	8.00	169	155	440	560	-33	-5	218	275	1999	-839	1999	-839
25	45.5	588	8.17	183	160	475	615	-35	+3	239	333	2091	-860	2091	-860
26	46.4	600	8.33	206	180	510	660	-35	0	259	392	2165	-884	2165	-884
27	47.4	612	8.50	235	190	555	740	-38	61	279	477	2261	-902	2261	-902
28	48.3	624	8.67	333	215	605	790	-35	83	295	-	2224	-2092	2224	-2092

TABLE A.12 BEAM 127





LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS IN. $\times 10^3$		REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
					EAST	WEST	1	2	3	4	5	6
0	-	6.5	0.09	-	0	0	0	0	0	0	0	0
1	-	48	0.67	-	10	10	-4	-1	0	0	25	-33
2	-	96	1.33	-	15	25	-1	-1	-3	1	59	-67
3	-	144	2.00	-	20	40	-2	-1	-4	1	94	-103
4	-	192	2.67	-	25	60	-1	-1	-5	-2	131	-140
5	-	240	3.33	-	35	85	-1	-1	-6	-3	172	-182
6	-	264	3.67	-	40	100	-1	-1	-7	-3	193	-203
7	-	288	4.00	-	45	110	-1	0	-7	-4	216	-227
8	-	312	4.33	-	50	125	-1	0	-7	-8	237	-249
9	-	336	4.67	-	50	135	-1	0	-7	-12	264	-277
10	-	360	5.00	-	60	155	+1	0	-8	-16	290	-300
11	-	384	5.33	-	60	175	0	0	-10	-21	319	-330
12	-	408	5.67	-	70	190	0	0	-11	-25	351	-354
13	-	432	6.00	-	75	215	0	6	-10	-15	385	-385
14	-	456	6.33	-	80	230	2	-3	-4	-8	415	-410
15	-	480	6.67	-	90	255	1	-2	0	0	447	-442
16	-	504	7.00	-	95	285	1	-3	3	4	480	-470
17	-	528	7.33	-	105	315	0	0	6	18	517	-514
18	-	552	7.67	-	115	340	2	-6	15	23	559	-543
19	-	576	8.00	-	120	375	2	-6	25	45	621	-590
20	-	600	8.33	-	130	410	1	-7	40	60	679	-626
21	-	624	8.67	-	145	455	0	-8	61	73	833	-684
22	-	648	9.00	-	160	505	0	-13	92	82	942	-720
23	-	672	9.33	-	180	575	-1	-17	123	73	1070	-767
24	-	696	9.67	-	200	650	-1	-20	160	64	1200	-814
25	-	706	9.82	-								

TABLE A.13 BEAM 201





LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS IN. $\times 10^3$			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
					EAST	CENTER	WEST	1	2	3	4	5	6
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0
1	13.0	72	1.00	0	5	10	15	6	4	8	2	50	-48
2	21.7	120	1.67	6	10	25	30	4	0	5	0	80	-78
3	26.0	144	2.00	10	15	30	40	2	2	6	0	98	-97
4	30.3	168	2.33	15	20	40	50	2	0	0	-5	113	-112
5	34.7	192	2.67	18	20	45	55	0	0	1	-6	132	-134
6	39.0	216	3.00	25	25	55	65	0	-4	1	-10	147	-148
7	43.3	240	3.33	29	25	60	75	-2	-4	-1	-15	167	-167
8	47.7	264	3.67	35	30	70	90	+4	-6	-3	-20	184	-183
9	52.0	288	4.00	42	35	80	100	6	-7	-7	-29	204	-204
10	56.3	312	4.33	47	40	90	115	16	-5	-8	-36	223	-221
11	60.7	336	4.67	58	45	100	135	30	-2	-6	-41	252	-248
12	65.0	360	5.00	64	50	115	150	45	-1	-6	-46	270	-266
13	69.4	384	5.33	74	50	130	170	64	+2	-5	-31	298	-290
14	73.7	408	5.67	86	60	150	200	131	5	-6	0	321	-310
15	78.0	432	6.00	103	65	170	225	168	10	-10	131	355	-339
16	82.4	456	6.33	126	70	195	260	284	15	-19	224	386	-363
17	86.7	480	6.67	154	85	220	285	352	21	-26	328	436	-398
18	91.0	504	7.00	186	90	250	325	584	38	-29	470	504	-423
19	95.4	528	7.33	229	100	275	360	858	64	-15	635	588	-453
20	99.7	552	7.67	296	120	325	430	1045	47	+194	792	733	-460
21	104.0	576	8.00	352	135	370	490	1140	52	304	933	820	-482
22	108.4	600	8.33	460	160	445	595	1182	956	487	1078	1020	-397
23	110.5		8.50										

TABLE A.14 BEAM 202



LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. x 10 <sup>6</sup>	DEFLECTIONS			REINFORCEMENT STRAINS					
					IN. x 10 <sup>3</sup>			MICRO INCHES PER INCH					
					EAST	CENTER	WEST	GAUGE					
					1	2	3	4	5	6			
0	0	6.5	0.09	0	0	0	0	0		0			
1	19.5	48	0.67	14	5	5	15	0	0	2			
2	29.2	72	1.00	21	10	15	20	0	-4	1			
3	39.0	96	1.33	32	15	20	25	0	-4	3			
4	48.7	120	1.67	42	15	25	35	-3	-6	7			
5	53.6	132	1.83	47	15	30	40	-4	-5	9			
6	58.5	144	2.00	53	20	35	45	-3	-5	11			
7	63.4	156	2.17	60	15	40	45	-5	-2	19			
8	68.2	168	2.33	65	20	45	50	-4	-2	21			
9	73.1	180	2.50	72	20	50	55	-5	0	30			
10	78.0	192	2.67	81	25	50	60	-2	8	33			
11	82.9	204	2.83	86	25	50	65	-3	10	42			
12	87.7	216	3.00	97	30	55	75	-4	15	54			
13	92.6	228	3.17	108	25	60	75	-2	23	68			
14	97.5	240	3.33	115	30	65	80	0	27	71			
15	102.4	252	3.50	129	35	75	85	3	39	103			
16	107.2	264	3.67	144	35	75	90	13	56	114			
17	112.1	276	3.83	168	40	85	105	23	77	95			
18	117.0	288	4.00	200	45	95	115	177	100	131			
19	121.9	300	4.17	253	45	105	130	397	200	667			
20	126.7	312	4.33	322	50	120	145	730	662	954			
21	128.7	317	4.40	393	55	130	160	1050	1078	1058			
22	130.6	322	4.47	456	55	135	170	1227	1328	1168			
23	132.6	326	4.53	511	55	145	175	1265	1465	1255			
24	134.5	331	4.60	565	55	155	185	1270	1584	1334			
25	136.5	336	4.67	610	60	155	190	1296	1696	1728			
26	138.4	341	4.73	668	65	165	205	1266	1797	1816			
27	140.4	346	4.80	729	70	170	210	1250	1904	1916			
28	142.3	350	4.87	789	70	185	220	1156	1965	2032			
29	144.3	355	4.93	861	70	190	230	1100	2040	2138			
30	146.2	360	5.00	932	75	200	240	1174	2106	2226			
31	148.2	365	5.07	1206	80	230	265	1390	2310	2361			
32	143.7	365	5.07	1588	80	280	295	3110	2228	2308			

TABLE A.15 BEAM 203





LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN.KIP	SHEAR KIP	TWIST RAD/IN. x 10 <sup>6</sup>	DEFLECTIONS			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE							
					EAST	CENTER IN. x 10 <sup>3</sup>	WEST	1	2	3	4	5	6		
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0	0	0
1	17.3	24	0.33	14	0	0	5	-2	1	1	-2	11	-15	11	-15
2	34.7	48	0.67	32	5	10	10	-4	5	9	-2	28	-4	28	-4
3	52.0	72	1.00	53	5	10	20	-5	13	17	0	41	-57	41	-57
4	60.7	84	1.17	61	10	15	20	-8	19	21	0	50	-67	50	-67
5	69.3	96	1.33	75	5	20	20	-5	23	31	3	54	-77	54	-77
6	78.0	120	1.67	88	10	25	25	0	34	49	12	78	-100	78	-100
7	86.7	120	1.67	101	5	25	30	11	45	64	20	76	-100	76	-100
8	95.3	132	1.83	118	0	25	35	18	60	83	30	90	-119	90	-119
9	104.0	144	2.00	136	0	25	35	36	78	112	47	102	-125	102	-125
10	112.7	156	2.17	160	0	30	45	118	109	131	77	117	-128	117	-128
11	121.3	168	2.33	206	5	35	50	385	201	157	125	136	-132	136	-132
12	124.8	173	2.40	236	0	40	60	545	300	170	167	151	-130	151	-130
13	128.2	178	2.47	268	0	40	65	632	443	184	216	169	-125	169	-125
14	131.7	182	2.53	306	0	50	65	729	649	196	311	197	-111	197	-111
15	135.2	187	2.60	400	0	55	75	900	969	236	558	221	-100	221	-100
16	138.6	192	2.67	533	5	65	90	705	1245	466	716	257	-88	257	-88
17	140.4	194	2.70	635	5	70	95	620	1357	594	848	311	-62	311	-62
18	142.1	197	2.73	701	5	75	100	592	1410	714	910	355	-47	355	-47
19	143.8	199	2.77	767	5	75	105	560	1430	804	966	404	-16	404	-16
20	145.6	202	2.80	828	10	80	105	554	1366	883	1020	461	+20	461	+20
21	147.3	204	2.83	890	10	80	115	540	1302	961	1068	512	60	512	60
22	149.0	206	2.87	960	10	90	120	537	1244	1063	1107	597	115	597	115
23	150.8	209	2.90	1046	10	90	125	625	1204	1200	1160	784	203	784	203
24	152.5	211	2.93	1174	10	100	130	689	1170	1383	1215	1190	332	1190	332
25	154.2	214	2.97	1375	15	110	145	734	1189	1561	1271	1756	483	1756	483
26	145.9	216	3.00	1701	15	125	160	769	1192	1653	1286	2850	711	2850	711
27	145.9	216	3.00	1917	25	130	175	780	1200	1672	1280	3748	794	3748	794

TABLE A.16 BEAM 204





LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
					EAST	CENTER	WEST	1	2	3	4	5	6
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0
1	19.5	12	0.17	15	0	5	5	-7	-4	1	1	3	0
2	39.0	24	0.33	38	-5	5	5	-8	-4	10	9	13	-11
3	46.8	29	0.40	49	-5	5	0	-5	-1	12	11	17	-17
4	54.6	34	0.47	58	-5	5	0	0	0	18	20	20	-20
5	62.4	38	0.53	69	-5	10	5	7	3	20	27	20	-15
6	70.2	43	0.60	81	-10	5	5	30	13	32	34	36	-22
7	78.0	48	0.67	94	-10	5	5	66	22	45	48	32	-28
8	85.8	53	0.73	110	-10	10	10	105	50	65	60	36	-78
9	93.6	58	0.80	126	-15	5	5	141	92	89	80	43	-39
10	101.4	62	0.87	150	-15	5	10	160	160	152	125	50	-40
11	109.2	67	0.93	189	-15	10	10	555	230	212	183	60	-26
12	117.0	72	1.00	256	-20	10	15	1080	314	282	269	81	-5
13	120.9	74	1.03	328	-20	15	20	1362	635	343	302	110	+86
14	124.8	77	1.07	443	-10	20	25	1939	1150	596	356	148	280
15	128.7	79	1.10	558	-15	20	25	1861	1459	1202	403	219	435
16	132.6	82	1.13	703	-10	25	30	1850	1558	1498	658	296	581
17	136.5	84	1.17	940	-15	30	40	3410	2000	1876	1150	544	741
18	139.9	86	1.20	1311	-10	40	50	4950	2085	2749	1351	1140	1114
19	139.9	86	1.20	1646	-10	45	60	8980	655	-	273	432	236
20	139.9	86	1.20	2015	-5	55	75	-	-720	-	-876	-328	-689

TABLE A.17 BEAM 205



LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN.  x 10 <sup>6</sup>	DEFLECTIONS			REINFORCEMENT STRAINS					
					IN. x 10 <sup>3</sup>			MICRO INCHES PER INCH					
					EAST	CENTER	WEST	GAUGE					
							1	2	3	4	5	6	
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	
1	10.2	120	1.67	10	25	30	-10	-12	-1	0	81	-79	
2	13.0	168	2.33	11	45	45	-9	-11	-1	0	125	-110	
3	16.7	216	3.00	17	55	65	-2	-15	-9	-1	170	-141	
4	18.6	240	3.33	18	65	75	-3	-11	-1	-1	199	-169	
5	20.4	264	3.67	19	70	85	-2	-11	-3	-3	220	-182	
6	22.3	288	4.00	24	80	95	-2	-11	-5	-3	251	-201	
7	24.1	312	4.33	28	90	105	0	-15	-10	-9	280	-220	
8	26.0	336	4.67	29	100	115	0	-18	-10	-9	311	-241	
9	27.9	360	5.00	33	115	135	5	-18	-10	-9	340	-260	
10	29.7	384	5.33	39	125	145	5	-15	-9	-5	380	-285	
11	31.6	408	5.67	42	140	170	10	-19	-9	-10	410	-305	
12	33.4	432	6.00	46	155	185	11	-18	-11	-10	459	-335	
13	35.3	456	6.33	51	170	210	19	-18	-19	-10	500	-355	
14	37.2	480	6.67	56	185	235	20	-18	-19	-10	555	-385	
15	39.0	504	7.00	63	205	255	28	-20	-20	+32	600	-405	
16	40.9	528	7.33	68	230	280	30	-20	-39	79	659	-440	
17	42.7	552	7.67	75	250	310	41	-20	-31	118	735	-468	
18	44.6	576	8.00	81	275	340	45	-19	-30	160	852	-501	
19	46.4	600	8.33	92	300	365	60	-25	+20	215	1011	-530	
20	48.3	624	8.67	100	325	405	65	-25	49	261	1160	-579	
21	50.2	648	9.00	111	360	445	83	-31	79	300	1271	-609	
22	52.0	672	9.33	125	395	500	91	-35	110	330	1430	-645	
23	53.9	696	9.67	140	450	565	112	-39	160	360	1589	-679	
24	55.7	714	9.92										

TABLE A.18 BEAM 207





LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS IN. $\times 10^3$		REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
					EAST	WEST	1	2	3	4	5	6
0	-	6.5	0.09	-	0	0	0	0	0	0	0	0
1	-	48	0.67	-	5	10	0	0	0	2	23	-39
2	-	96	1.33	-	10	20	0	2	0	4	50	-76
3	-	144	2.00	-	15	30	-3	-2	-3	1	84	-112
4	-	192	2.67	-	25	50	-2	0	0	0	115	-152
5	-	240	3.33	-	30	60	-4	-4	-4	-1	147	-198
6	-	288	4.00	-	35	80	-4	-4	+2	-4	182	-244
7	-	336	4.67	-	45	100	-6	-4	-2	-5	217	-293
8	-	384	5.33	-	55	115	-8	-8	-6	-6	256	-342
9	-	432	6.00	-	65	135	-10	-6	-6	-14	299	-402
10	-	480	6.67	-	75	155	-8	-7	-12	-18	343	-454
11	-	528	7.33	-	80	185	-9	-7	-14	-20	388	-521
12	-	576	8.00	-	95	215	-11	-11	-20	-27	435	-567
13	-	624	8.67	-	105	245	-16	-8	-22	-33	498	-644
14	-	672	9.33	-	115	275	-17	-17	-28	-25	555	-708
15	-	720	10.00	-	130	315	-16	-20	-28	+5	619	-772
16	-	744	10.33	-	140	335	-17	-22	-32	16	659	-820
17	-	768	10.67	-	150	355	-17	-27	-27	31	711	-850
18	-	792	11.00	-	160	375	-16	-26	-26	50	756	-910
19	-	816	11.33	-	160	395	-15	-18	-18	73	781	-937
20	-	840	11.67	-	170	415	-17	-32	-15	85	835	-994
21	-	864	12.00	-	180	440	-10	-26	-2	112	890	-1020
22	-	888	12.33	-	190	460	-12	-26	+6	144	961	-1068
23	-	912	12.67	-	200	490	-18	-30	8	160	1024	-1101
24	-	936	13.00	-	210	520	-14	-24	14	178	1108	-1153
25	-	960	13.33	-	225	560	-16	-28	27	198	1178	-1195
26	-	972	13.50	-	230	580	-16	-31	33	210	1229	-1224
27	-	984	13.67	-	235	600	-17	-35	40	214	1263	-1240
28	-	996	13.83	-	255	640	-16	-25	48	225	1350	-1280
29	-	1008	14.00	-	260	660	-17	-32	58	232	1391	-1298
30	-	1020	14.17	-	285	820	-20	-30	62	975	1355	-1263

TABLE A.19 BEAM 221





LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN.KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS			REINFORCEMENT STRAINS					
					EAST	CENTER $\text{IN.} \times 10^3$	WEST	1	2	3	4	5	6
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0
1	13.0	72	1.00	8	10	15	20	0	0	0	0	0	-59
2	21.7	120	1.67	17	20	30	40	0	0	-2	0	0	-108
3	30.3	168	2.33	32	25	45	55	2	9	1	5	5	-149
4	39.0	216	3.00	38	25	60	70	5	15	5	9	10	-200
5	43.3	240	3.33	39	30	70	80	10	19	5	10	10	-220
6	47.7	264	3.67	47	35	75	90	10	25	5	10	10	-248
7	52.0	288	4.00	54	40	85	100	11	31	10	11	11	-270
8	56.3	312	4.33	61	45	95	115	15	40	12	15	15	-300
9	60.7	336	4.67	65	50	105	125	21	49	20	19	19	-321
10	65.0	360	5.00	72	50	110	130	29	61	20	20	20	-350
11	69.4	384	5.33	82	55	125	145	40	73	30	25	25	-375
12	73.7	408	5.67	90	60	135	160	48	92	41	30	30	-405
13	78.0	432	6.00	100	65	145	180	69	109	59	30	30	-431
14	82.4	456	6.33	110	70	160	195	78	120	72	39	39	-459
15	86.7	480	6.67	118	75	175	210	90	132	102	45	45	-488
16	91.0	504	7.00	131	80	185	230	105	152	150	53	53	-513
17	95.4	528	7.33	147	90	205	250	149	200	200	100	100	-540
18	99.7	552	7.67	189	100	225	280	251	287	309	148	148	-570
19	104.0	576	8.00	192	105	240	305	365	341	481	260	260	-600
20	106.2	588	8.17	211	105	255	320	440	395	642	320	320	-618
21	108.4	600	8.33	231	110	270	335	520	439	759	360	360	-631
22	110.5	612	8.50	246	115	285	355	558	481	840	403	403	-651
23	112.7	624	8.67	261	115	295	370	668	562	911	439	439	-665
24	114.9	636	8.83	288	120	305	390	720	690	980	470	470	-681
25	117.0	648	9.00	324	125	315	405	810	1090	1135	510	510	-681
26	119.2	660	9.17	368	135	335	430	891	1449	1232	560	560	-700
27	121.4	672	9.33	406	140	350	445	1020	1602	1305	612	612	-710
28	123.5	684	9.50	440	145	365	470	1121	1695	1369	661	661	-730
29	125.7	696	9.67	476	145	380	485	1280	1765	1435	715	715	-840
30	127.9	708	9.83	544	155	395	510	1353	1865	1504	763	763	-835
31	130.0	720	10.00	590	170	420	550	1510	1808	1845	960	960	-782
32	132.2	732	10.17	644	180	455	590	1525	1761	1790	965	965	-789
33	134.4	744	10.33	685	190	480	635	1560	1950	1790	1012	1012	-792
34	136.5	756	10.67	644	265	725	970	1360	1990	1555	1000	1000	-865

TABLE A.20 BEAM 222



LOAD STAGE	TORQUE IN. KIP	BENDING MOMENT IN. KIP	SHEAR KIP	TWIST RAD/IN. x 10 <sup>6</sup>	DEFLECTIONS IN. x 10 <sup>3</sup>			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE									
					EAST	CENTER	WEST	1	2	3	4	5	6				
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	19.4	48	0.67	13	5	10	10	10	-5	-5	-2	20	-38				
2	29.2	72	1.00	25	10	15	15	15	0	-10	-5	31	-53				
3	39.0	96	1.33	35	10	20	20	20	0	-10	-5	49	-75				
4	48.7	120	1.67	46	10	25	35	35	0	-10	-8	60	-95				
5	53.6	132	1.83	51	15	30	35	35	0	-9	-8	69	-108				
6	58.5	144	2.00	58	15	35	40	40	0	-9	-8	75	-118				
7	63.3	156	2.17	65	15	35	40	40	0	-9	0	82	-128				
8	68.2	168	2.33	71	15	40	50	50	9	-2	0	91	-138				
9	73.1	180	2.50	79	20	40	55	55	10	-2	4	99	-151				
10	78.0	192	2.67	86	15	45	55	55	21	-1	10	101	-161				
11	82.9	204	2.83	93	20	50	60	60	30	+4	20	110	-171				
12	87.7	216	3.00	101	25	55	65	65	57	12	29	119	-180				
13	92.6	228	3.17	113	25	60	65	65	78	19	32	129	-190				
14	97.5	240	3.33	119	25	65	75	75	119	25	39	132	-201				
15	102.4	252	3.50	135	30	70	80	80	152	41	52	142	-211				
16	107.2	264	3.67	146	30	70	85	85	232	59	68	150	-220				
17	112.1	276	3.83	165	30	75	90	90	310	112	87	160	-230				
18	116.9	288	4.00	183	30	85	100	100	538	152	98	170	-238				
19	121.9	300	4.17	271	30	95	110	110	-	392	182	200	-218				
20	126.7	312	4.33	314	30	100	120	120	1129	820	225	231	-172				
21	131.6	324	4.50	475	30	110	135	135	1530	1565	941	310	-31				
22	133.6	329	4.57	576	35	120	140	140	1630	1860	1170	351	+110				
23	136.5	336	4.67	704	35	130	155	155	1720	2020	1290	962	310				
24	138.4	341	4.73	799	35	130	170	170	1808	-	1370	-80	-760				
25	140.4	346	4.80	921	35	135	175	175	725	-	170	0	-625				
26	142.3	350	4.87	1054	40	150	190	190	-388	-	-996	-110	-420				
27	144.3	355	4.93	1232	40	160	205	205	-300	-	-910	-710	-1460				
28	146.2	360	5.00	1506	40	170	220	220	-340	-	-720	-300	-1140				
29	148.2	365	5.07	1913	40	185	255	255	-250	-	-400	+630	-1080				
30	148.2	367	5.10	2358	50	195	290	290	+1620	-	-60	1480	-1220				

TABLE A.21 BEAM 223





LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN.KIP	SHEAR KIP	TWIST RAD/IN. $\times 10^6$	DEFLECTIONS IN. $\times 10^3$		REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE					
					EAST	WEST	1	2	3	4	5	6
0	0	6.5	0.09	0	0	0	0	0	0		0	0
1	17.3	24	0.33	22	0	5	-10	-10	-15		2	-15
2	34.7	48	0.67	38	0	10	-20	-18	-22		10	-37
3	52.0	72	1.00	56	5	20	-14	-16	-20		36	-53
4	60.7	84	1.17	65	10	25	-13	-20	-30		43	-64
5	69.3	96	1.33	78	5	30	-10	-12	-30		50	-80
6	78.0	108	1.50	89	5	35	-6	-12	-30		58	-85
7	86.7	120	1.67	103	5	40	-2	0	-33		60	-96
8	95.3	132	1.83	115	5	35	+4	18	-34		72	-102
9	104.0	144	2.00	133	5	40	11	40	-33		84	-106
10	112.7	156	2.17	151	5	50	144	82	-33		99	-100
11	121.3	168	2.33	185	10	50	314	146	-38		106	-91
12	124.8	173	2.40	203	5	55	426	187	-24		118	-84
13	128.2	178	2.47	218	5	60	520	224	-13		122	-70
14	131.7	182	2.53	240	5	55	646	416	+2		130	-20
15	135.2	187	2.60	290	5	55	840	980	14		142	+235
16	138.6	192	2.67	382	5	60	1125	1338	426		204	384
17	140.4	194	2.70	457	0	60	1315	1544	576		253	460
18	142.1	197	2.73	526	0	65	1400	2016	720		250	486
19	143.8	199	2.77	571	-5	70	1485	2562	810		255	486
20	145.6	202	2.80	642	-5	65	1496	3420	912		260	554
21	147.3	204	2.83	697	-5	70	1520	4690	1020		270	636
22	149.0	206	2.87	756	-5	75	1528	6310	1112		278	734
23	150.8	209	2.90	811	-10	75	1542	7360	1172		294	814
24	152.5	211	2.93	864	-10	75	1582	7554	1223		300	920
25	154.2	214	2.97	929	-10	80	1703	7980	1275		328	1024
26	156.0	216	3.00	1008	-10	85	456	-	1125		-792	-10
27	157.7	218	3.33	1206	-20	85	422	-	1144		-662	+300
28	159.4	221	3.07	1383	-20	90	450	-	1172		-426	778
29	160.2	223	3.10	1701	-25	100	828	-	1181		+114	1522

TABLE A.22 BEAM 224





LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN.KIP	SHEAR KIP	TWIST RAD/IN. x 10 <sup>6</sup>	DEFLECTIONS IN. x 10 <sup>3</sup>			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE								
					EAST	CENTER	WEST	1	2	3	4	5	6			
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0	0	0	0
1	19.5	12	0.17	17	0	0	0	-3	-2	-8	-3	2	-2	-2	-8	-2
2	39.0	24	0.33	40	0	0	0	-6	0	-5	-4	4	0	-4	-8	-8
3	46.8	29	0.40	49	0	5	5	-6	0	0	-5	5	0	-5	-11	-11
4	54.6	34	0.47	60	5	5	5	-6	0	3	-5	8	0	-5	-15	-15
5	62.4	38	0.53	69	5	5	5	-4	2	7	-2	10	2	-2	-15	-15
6	70.2	43	0.60	83	-5	5	5	+10	9	17	+3	16	9	+3	-15	-15
7	78.0	48	0.67	96	-5	5	5	30	15	20	16	16	15	16	-23	-23
8	85.8	53	0.73	108	-5	5	5	60	20	32	28	16	20	28	-24	-24
9	93.6	58	0.80	125	-5	0	0	85	31	41	45	25	31	45	-20	-20
10	101.4	62	0.87	144	-10	0	0	181	70	79	65	38	70	65	-20	-20
11	109.2	67	0.93	218	-15	-5	0	252	97	130	95	16	97	95	-32	-32
12	117.0	72	1.00	351	-25	-5	-5	414	779	180	175	20	779	175	-28	-28
13	120.9	74	1.03	447	-25	-10	-5	502	1026	585	241	135	1026	241	+33	+33
14	124.8	77	1.07	594	-30	-15	-5	611	1230	998	737	397	1230	737	158	158
15	128.7	79	1.10	675	-35	-20	-5	750	1355	1102	884	477	1355	884	260	260
16	132.6	82	1.13	811	-35	-30	-5	830	1586	1253	1039	644	1586	1039	400	400
17	136.5	84	1.17	976	-40	-30	-10	836	3322	1335	1163	870	3322	1163	556	556
18	140.4	86	1.20	1192	-50	-45	-15	610	-	1218	86	0	-	86	-436	-436
19	142.1	88	1.23	1982	-55	-70	-15	402	-	262	236	510	-	236	-77	-77
20	142.1	88	1.23	2519	-55	-90	-25	466	-	240	203	-55	-	203	-52	-52

TABLE A.23 BEAM 225



LOAD STAGE	TORQUE IN.KIP	BENDING MOMENT IN.KIP	SHEAR KIP	TWIST RAD/IN. x 10 <sup>6</sup>	DEFLECTIONS			REINFORCEMENT STRAINS MICRO INCHES PER INCH GAUGE							
					IN. x 10 <sup>3</sup>										
					EAST	CENTER	WEST	1	2	3	4	5	6		
0	0	6.5	0.09	0	0	0	0	0	0	0	0	0	0	0	0
1	10.1	120	1.67	6	10	25	35	-4	-4	2	-2	75	-96	-155	-198
2	14.9	192	2.67	14	25	45	55	-5	-3	-2	-3	122	-155	-198	-244
3	18.6	240	3.33	15	30	60	75	-7	-2	-1	-3	158	-198	-244	-292
4	22.3	288	4.00	21	35	80	100	-8	-1	-2	-7	190	-244	-292	-341
5	26.0	336	4.67	25	45	95	120	-10	-3	-3	-9	227	-292	-341	-394
6	29.7	384	5.33	32	50	115	140	-5	-5	-1	-13	263	-341	-394	-446
7	33.4	432	6.00	39	60	135	170	-7	-3	-4	-19	306	-394	-446	-505
8	37.2	480	6.67	46	70	160	195	-6	-1	-2	-28	342	-446	-505	-567
9	40.9	528	7.33	53	80	185	230	-5	-1	-2	-33	390	-505	-567	-638
10	44.6	576	8.00	63	90	210	270	-4	0	0	-28	429	-567	-638	-706
11	48.3	624	8.67	75	105	245	310	-1	0	0	-5	493	-638	-706	-784
12	52.0	672	9.33	89	115	285	355	+24	7	21	+105	548	-706	-784	-819
13	55.7	720	10.00	104	135	320	405	64	25	50	291	618	-784	-819	-872
14	57.6	744	10.33	117	140	340	435	100	44	74	367	695	-819	-872	-925
15	59.4	768	10.67	129	150	365	460	113	54	100	460	771	-872	-925	-955
16	61.3	792	11.00	143	155	390	495	132	70	165	548	842	-925	-955	-998
17	63.2	816	11.33	156	165	415	525	152	88	239	629	925	-955	-998	-1051
18	65.0	840	11.67	175	175	440	555	250	132	391	718	1000	-998	-1051	-1090
19	66.9	864	12.00	204	185	470	595	402	213	531	818	1072	-1051	-1090	-1116
20	68.7	888	12.33	246	195	500	620	660	435	626	902	1151	-1090	-1116	-1171
21	70.6	912	12.67	268	205	525	655	740	502	696	979	1240	-1116	-1171	-1216
22	72.4	936	13.00	294	220	565	710	821	566	795	1084	1338	-1171	-1216	-1232
23	74.3	960	13.33	336	245	645	835	902	627	879	1400	1485	-1216	-1232	-1144
24	75.2	972	13.50	413	280	730	930	958	709	1047	1848	1564	-1232	-1144	
25	76.2	984	13.67	617	355	990	1265	945	743	1170	2251	1336	-1144		

TABLE A.24 BEAM 227



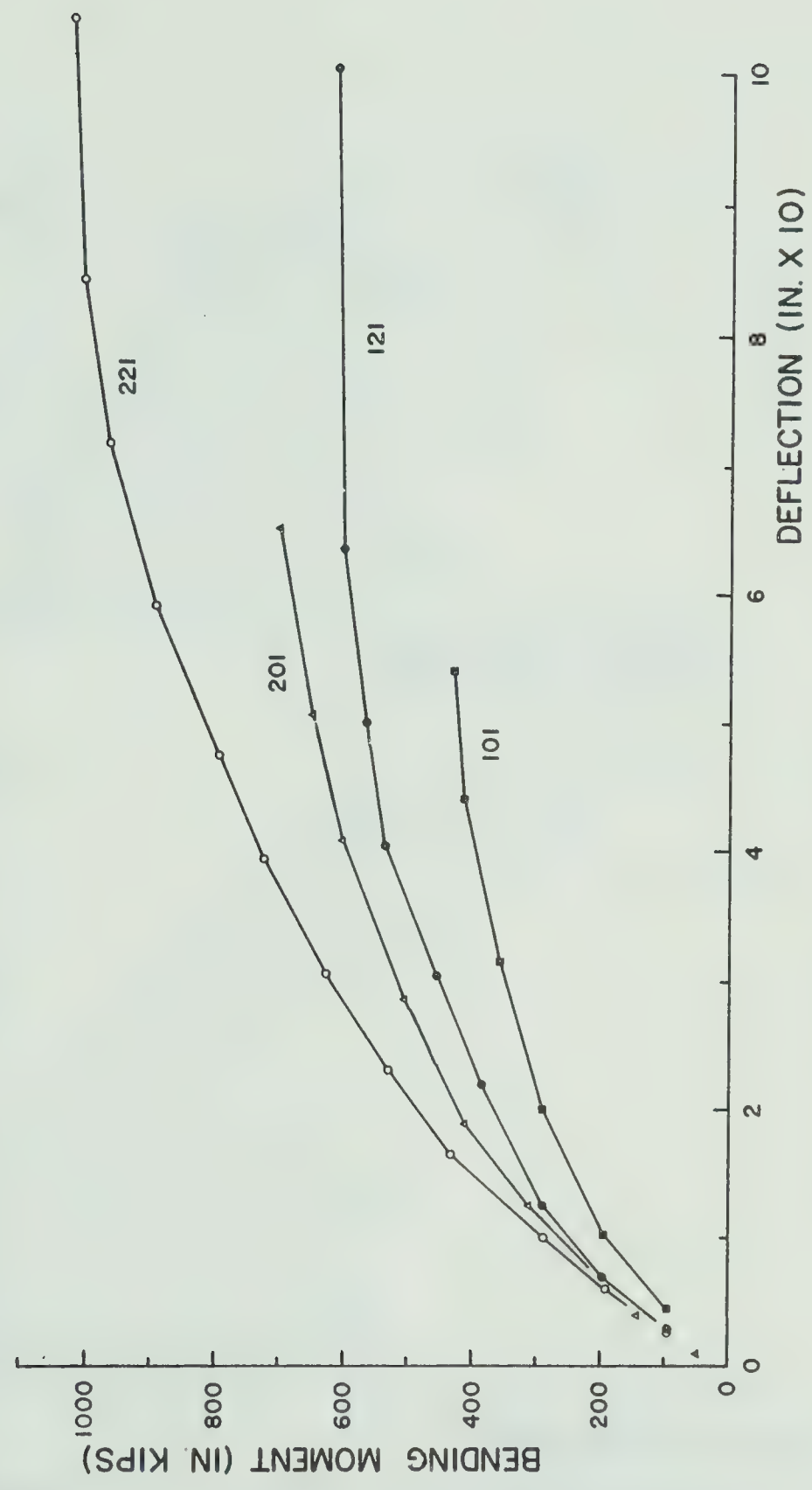


FIGURE A.5 MOMENT DEFLECTION CURVES





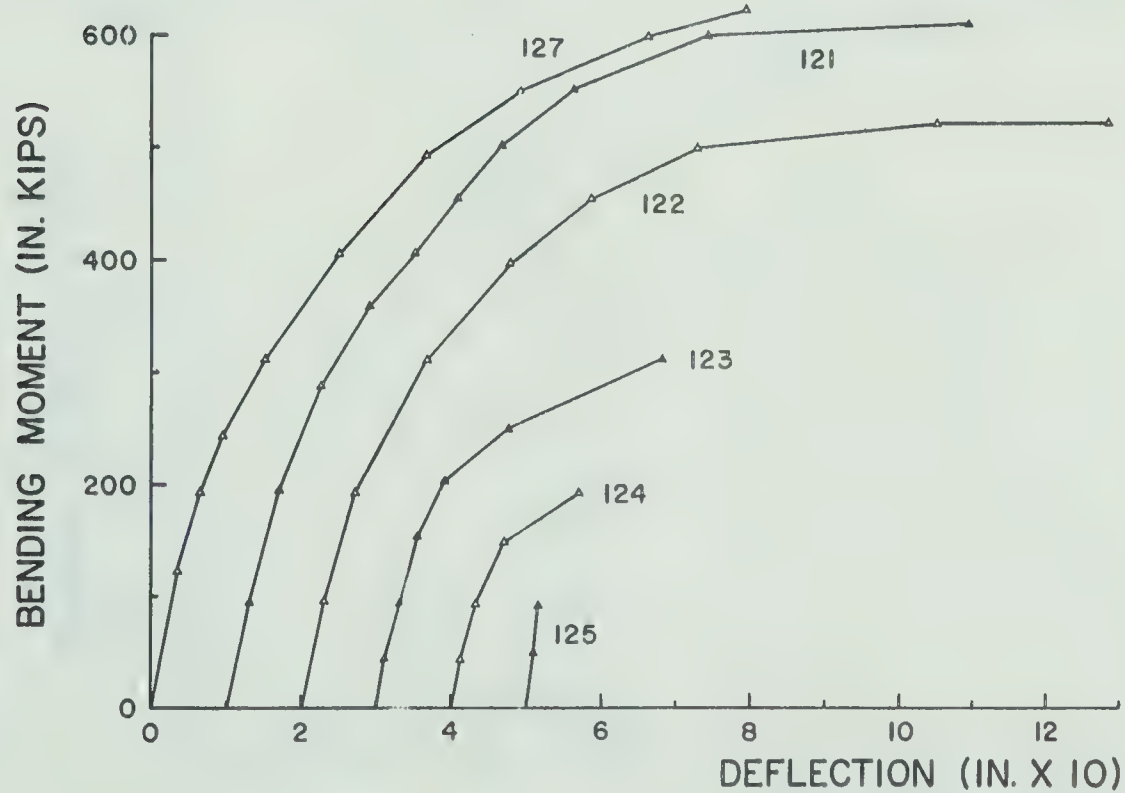
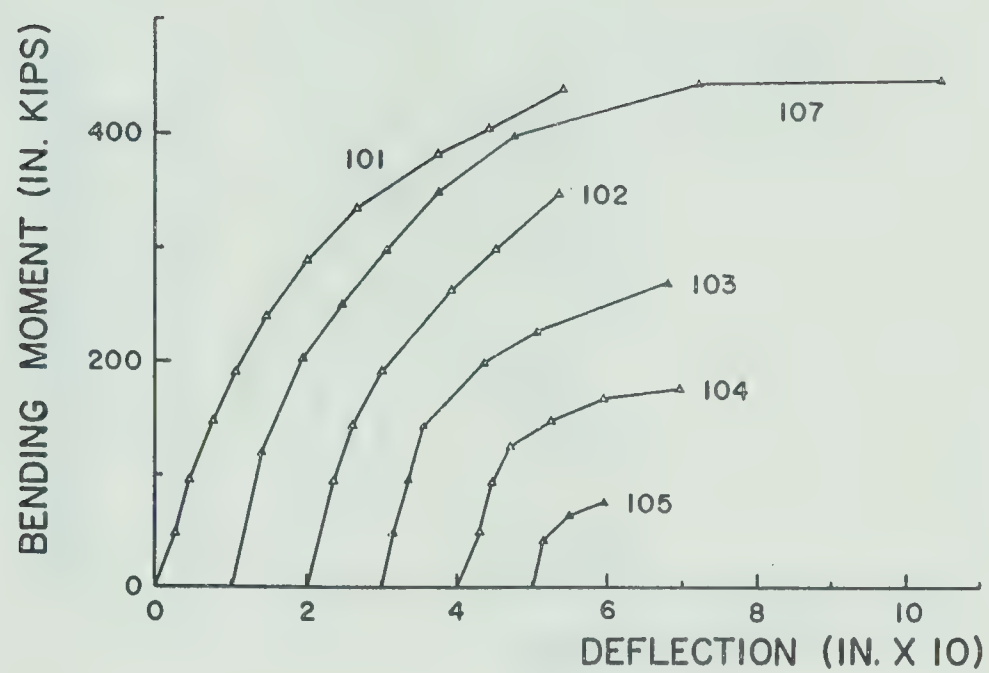


FIGURE A.6 MOMENT DEFLECTION CURVES



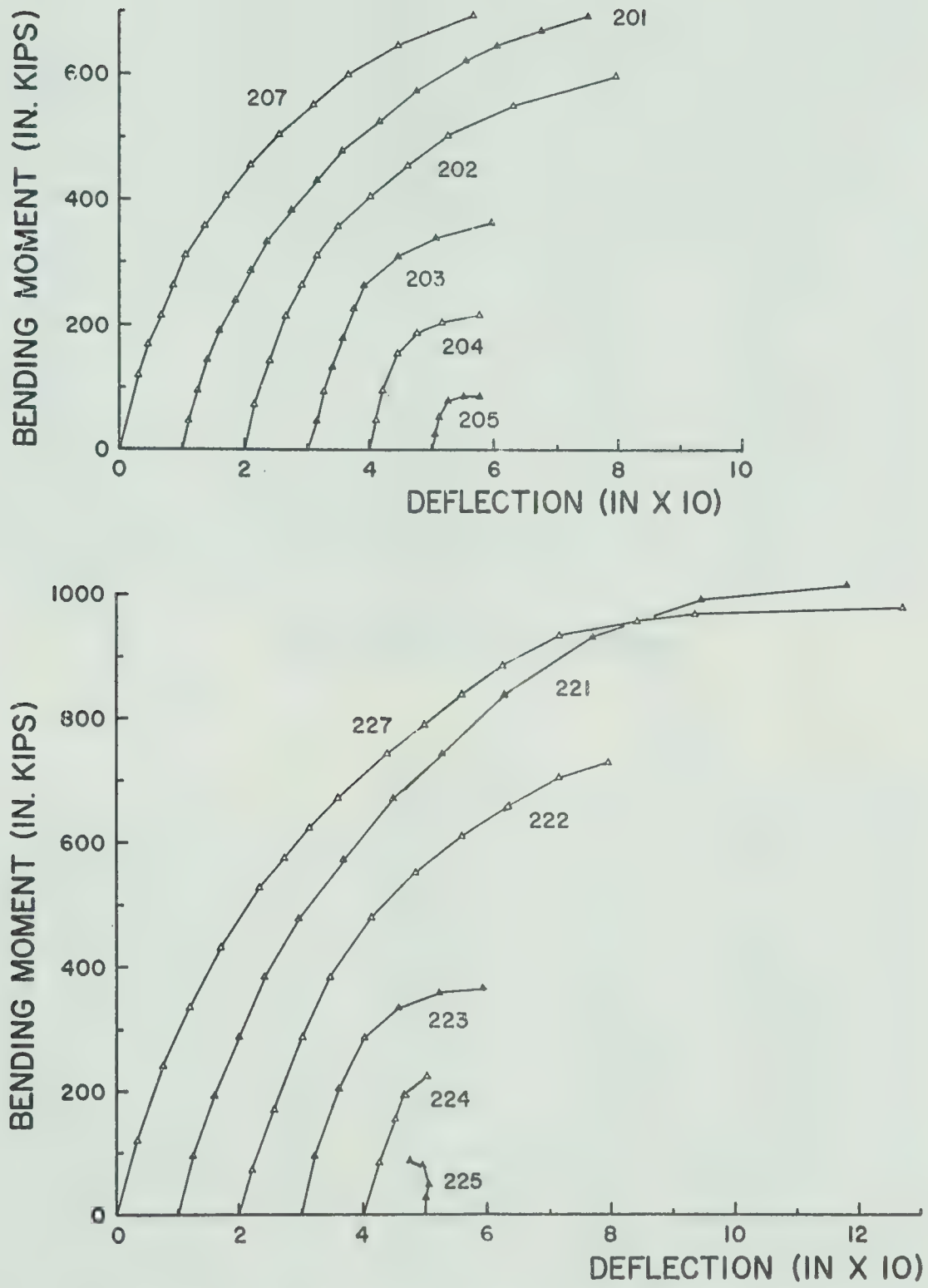


FIGURE A.7 MOMENT DEFLECTION CURVES



APPENDIX B

PHOTOGRAPHS OF SPECIMENS







NO PHOTOGRAPH AVAILABLE

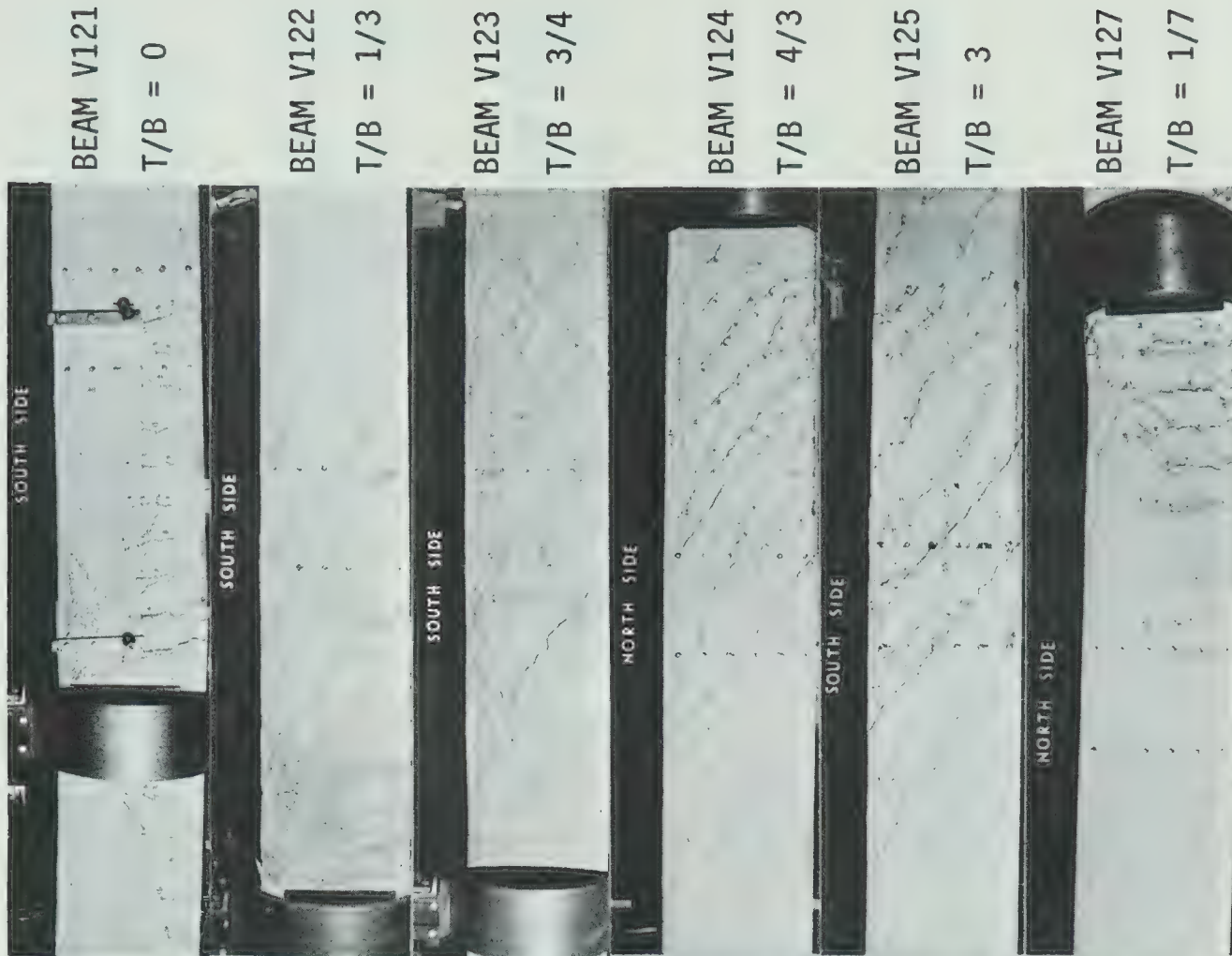
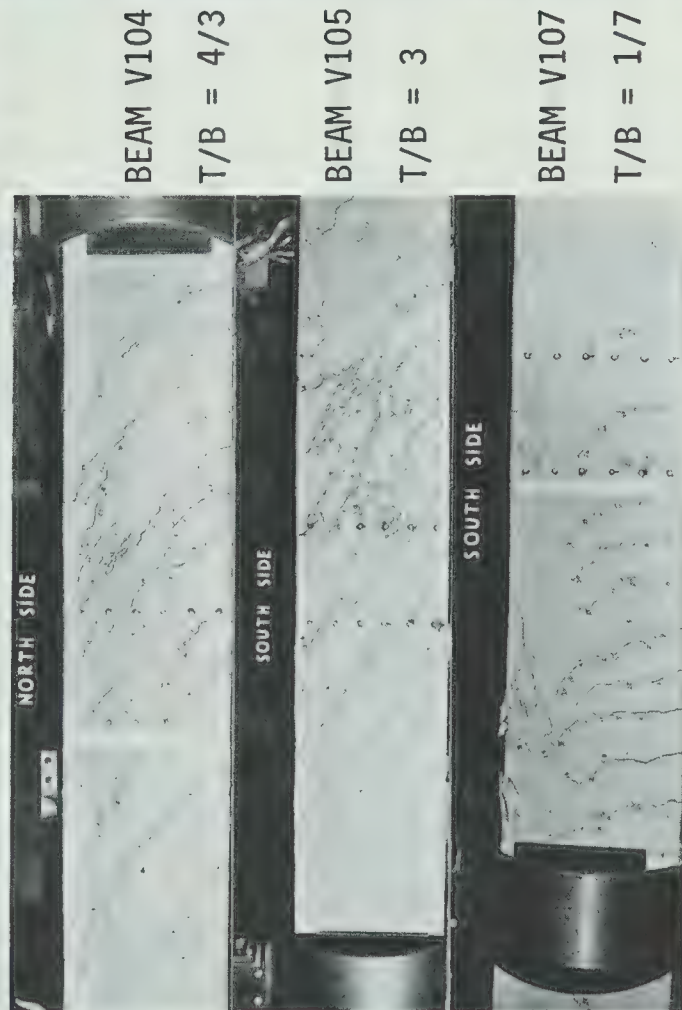


FIGURE B.1 CRACK PATTERNS (BEAMS V101 to V107) AND (BEAMS V121 to 127)



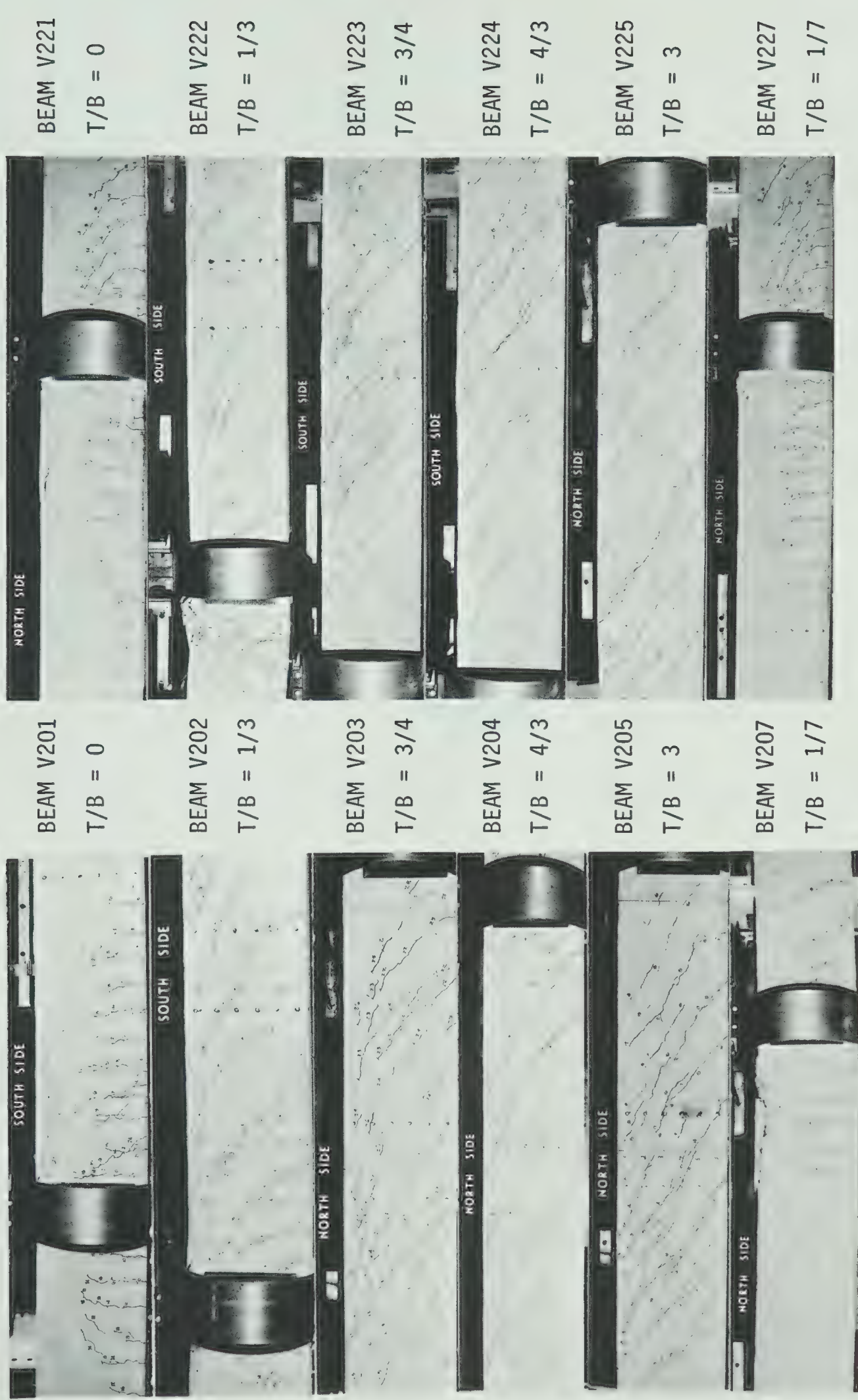


FIGURE B.2 CRACK PATTERNS (BEAMS V201 to V207) AND (BEAMS V221 to V227)





## APPENDIX C

### NOTATION





## NOTATION

- $f'_c$  = compressive strength of concrete determined by tests on 6 x 12 in. concrete cylinders
- $f'_{sp}$  = tensile strength of concrete as determined by tensile splitting tests
- $f_{y1}$  = yield strength of longitudinal reinforcement
- $f_{yt}$  = yield strength of lateral reinforcement
- $P_l$  = longitudinal steel percentage
- $P_t$  = transverse steel percentage
- $T_u$  = ultimate torsional moment under combined loading
- $T_{u0}$  = ultimate torsional moment under pure torsion
- $M_u$  = ultimate flexural moment at failure plane section under combined loading
- $M_{u0}$  = ultimate flexural moment under pure flexure
- $V_u$  = ultimate transverse shear in gauge length at failure under combined loading
- $V_{u0}$  = ultimate transverse shear in gauge length at failure under bending and shear without the presence of torsional loading

















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